Thrust on the walls is a horizontal force and has only an indirect connection with the downward weight (force) of the vault, and results from the manner in which the outer halfvaults are supported. It will be helpful to think of the weight of each vault unit as acting straight downward through the center of gravity of the unit, and at first to disregard the fact that the unit is trying to fall down as a whole. ~~Tx~~ Let us assume that we are giants, or better, that we are observing the behavior of scale models, and that at first we are going to support them, not on their bases, but on wire axes passing through holes bored longitudinally through sections of ~~our~~ each model, (which are) cut to a convenient length. Whatever such a piece does, the vault unit under discussion would do as a whole, unless supported by partitions, end walls, pillasters, cross-beams &c., which we will rule out of the discussion.

Fig. 1 is a cross section of a model of ~~a~~ balanced double-half vaults, such as are found everywhere over interior walls, serving rooms on either side of the wall. It is supported on a springy wire rising from ~~the~~ a base on the table, and bent at a right angle to pass through a hole in the model. The wire is stiff enough to support the dead weight of any of the models we shall put on it, but will bend if we push against it side-wise. The hole passes through the center of gravity. If the position of figure 1 is changed to that of Fig 2 (or any other) the vault will stay in the new position, because all of its weight operates through the center of gravity, which is also its point of support. It is free to revolve about the axis but does not do so because there is nothing to move it ; its weight, straight down, is met by an equal force straight upward through the wire. It might be well for dirt archeologists like the writer to remember that in his physics class he learned that whenever an object is in equilibrium, every force acting upon it must be met by an equal force acting in the opposite direction.

Now if we take the model off the wire axis and place it on its base on ~~in~~ a model (section of) wall, it will ~~xixxxxixx~~ not only stay put, but will resist any force tending to move it. It will resist being slipped side-wise across the top of the wall because of the friction caused by its weight (which in a real vault is ~~xxx~~ great). It is hardly necessary to remark that it will resist being lifted straight up by the amount of its weight. In addition, it will now resist~~s~~ being turned to the position of Figure 1. As soon as this is ~~attempted~~ done (Fig 4) the point (a) becomes a fulcrum. The corner cannot descend a little as it did in Fig 1 as compared with ~~opposed to~~ Fig 2 because the wall is in the ~~wall~~ way. Therefore the center of gravity has been raised as the whole mass is revolved about point (a) instead of about the center of gravity. This type of vault section not only has no tendency to fall off the wall, but actively insists that it stay where it is, despite the fact that its support is well below the center of gravity, which of course moves the center of garvity away from the hole,

Now let us slice off one of the half vaults, and put the model ~~The-center-of-gravit-has-moved~~ on the wire axis again, ~~holding it in the upright position~~ It will assume the position of Fig 5 because, being free to revolve on the axis, the weight, acting through the new center of gravity, pulls the latter to a point below the support. Now let us remove this vault section and place a corresponding wall model on ~~the wire~~ a rigid axis, passing the latter through the center of gravity of the wall, and adjust it to a vertical position. It is free to revolve about the axis ~~(in this case center of gravity also)~~ as did the vault section b in figures 1 and 2, but will stay where it is put since the axis passes through the center of gravity and the mass is a balanced unit, no matter what its position on the wire. We will ~~adjust~~ pass another ~~wire~~ rigid axis through the half vault of Fig 5 so that, when in a erect position, the base of the vault just touches the top of the wall, but with~~out~~ ~~any weight~~ all the weight supported by its own upper axis. Wall and vault are free to revolve, each on its own axis, provided one does not get in the way of the other. But a glance at Fig 6 will show that they do. For the half vault to take the position of Fig 5, its lower corner (a) must descend into the wall. For the wall to move counter-clockwise to allow such a movement of the vault, its corner (the same point a) must rise into the vault. It is clear therefore that this ~~xxx~~ combination will remain in equilbrium when the wire suports are removed and it is simply stood on the table (the previously prepared floor in a real Maya building). But the ~~xxx~~ situation is not the same as it was with balanced half-vaults (Fig 3). ~~Txx~~ Here the single half vault still <u>tends</u> to assume the position of Fig. 5, and tends to do so with a force equal to the total weight

The combination is in equilbirium, but the half-vault still tends to assume the position of Fig. 5; and it tends to do so with a force equal to the weight of the whole vault multiplied by the horizontal distance of the center of gravity from the supporting axis.
~~The weight is a downward pressure acting on a lever running from b (the axis of support) which is a fulcrum, to c, where the pressure is applied.~~
The axis (b) is a fulcrum, the horizontal distance to the vertical ~~xxxxx~~ line through the center of gravity is the lever (b-c), since the whole weight acts through this vertical line.

Now let us take two identical units, ~~ex~~ but shape the top of the wall and the bottom of the vault so that the movement indicated can take place. We will paste felt over each of these surfaces so that one cannot simply ~~ship~~

slip ~~myxthexother~~ across the other. The felt represents friction. ~~and~~ For the sake of simplicity we will assume that this operation has not changed our two centers of gravity. The vault will ~~not~~ now begin to assume the position of Fig 5. In doing so it will act like a cog on the wall, which will be moved to the position of Fig 7. This figure is ~~to~~ meant to illustrate the fact that in figure 6, ~~althoughxthexwallxsupportsxno~~ even if we eliminate the downward pressure of the corner(a), which it exerts on the same corner of the wall, (and the equal upward reaction of the wall,) there remains a horizontal side-force - the thrust - acting on the wall, so long as we do not eliminate traction between vault and wall. Fig. 6 is in ~~equxib~~ equilibrium not only because the wall is strong enough to prevent point (a) of the vault from moving downward (and the vault is strong enough to prevent the same point of the wall from moving upward: but also because ~~it~~ the wall is stiff enough to resist the motion of that point horizontally outward. Fig. 8 substitutes week spring wire like that of Figs 1 and 2 to illustrated the bending tendency of the outward as well as downward pressures at point a.

It is clear that if we take the combination of figure 6 off the wire axis and set it up on the table, it will stand as did figure 3. It is still impossible to tip the vault about point(a). The center of gravity, no longer ~~inxthexcenterxofxthe~~ over the center of the wall, is still over the wall. The vault will tip more easily in one direction than the other but in either case the center of gravity must be raised. The situation differs from Fig 3 however. The wall no longer merely resists downward pressure. It also resists an outward thrust. We can more easily examine the effect on the wall of this combination of forces ~~on the wall~~ in connection with more extreme examples of thrust.

If we change the form of our vault suitably, we can get its center of gravity immediately over the inner edge of the wall (point "a"). Here it is obvious that the state of equilbirium of the vaults of Figs 3 and 9 vanishes. The slightest horizontal push on the vertical side of the halfpvault will send it over (on a rotation about point a.) The center of gravity being immediately above ~~the fulcrum~~ point (a), ~~the~~ it is not raised by the tipping, but on the contrary immediately begins to fall. ~~The rest of the half vault will the center of gravity actually out over the room~~ AS soon as we make further changes so that the center of gravity is over the room, the weight of the half-vault itself does the tipping, and ~~it~~ (the vault) will fall of its own accord, at first rotating about point(A)as an axis. Here ~~as a force moment is in as of itself~~ The force behind this movement will be the weight of the half vault times the horizontal distance between axis and center of gravity, (cb, Fig 10), as in figure 6, but here the axis is a real one. Omitting allowances for factor of safety against masonry failure near the edge of the wall and vault, point a

(The lever actually in operation is ac, but in this position the leverage effect is the same as that of bc, at right angles to the line of force. W.)

in the model ~~xx~~ corresponds to the actual axis about whcih a full sized vault tends to fall. (Fig 11).

A.

Now it is obvious that such a half-vault must be supported at some point, in addition to its base. We will put it on a spring-wire axis. ~~xxxxxxxxx~~ First let us bring a horizontal force against the vault slope, at the level of the center of gravity, just suffient to keep the vault upright. (In real work such a force might be exerted by beams crossing from vault to vault).

(bc that is shortest distance between the downward force through the center of gravity and the upward force through the fulcrum at (a).)

The rotaional force to be met is ~~xxxx~~ weight times the line ab. ~~-x=a-- xxxx~~ ~~xxe. xxxxxxxxx~~

B

The horizontal force here necessary ~~will acts on the lever xx~~ acts directly through the center of gravity. ~~represented by line bc;~~ cd the shortest ~~vertical~~ distance between the same fulcrum and the horizontal line of our outside force. Our problem calls for an equal and opposite resisting force, no more. ~~and~~ it will therefore be Therefore, letting w equal the vault weight, and f the horizontal force, w x ab equals f. ~~x bc~~ cd. If we raise our support to the top, ~~bc becomes longer~~ (the lever cd ~~becomes bc plus cd~~ is introduced) and the necessary horizontal ~~force~~ force is much reduced.

again, the real lever is ce, but this is equivalent in effect to cd, the shortest distance ~~to the line of force, in this case horizontal~~ from the center of gravity to the line of force (this one being horizontal.)

?? R

The roof cap forms such a hori zontal support in complete buildings, and we have agreed to leave beams and other extra supports out of the discussion. As long as we do this we may simplify matter somewaht by noting that the effective lever on which the horizontal counter-thryst of the roof cap acts is equal to the vault height less the height of the center of gravity. (cd, c-e in our figure). The effective lever of the vault weight is ~~ab~~ the horizontal distance between the inner face of the wall and a line dropped from the center of gravity.

The application of the unequal horizontal forces in figures 12 and 13 has beent our spring-wire axis supports to the side in unequal degrees, To keep them straight we must introduce a horizontal force from the oppost direction. Here it would be well to remember that we are dealing ~~at~~ only with the horizontal ~~forces~~ components of the forces involved. The vault weight (~~down~~ downward) was one of the factors determing the amount of rotational force. ~~At the top where the line passing vertically through the center of gravity~~, This turning force is tangent to the circle of rotation, ~~that is horizontal at any point above the~~ about poibt(a) and in ~~the~~ Fig. 13 point w tends to go down as well as to the right. ~~The only way to isolate the horizontal forces is to take the half vault off its~~ But the vault is rigid and our horizontal force is transmitted ~~to~~ through it to the point d, were it meets the full rotational force, here entirely horizontal in the opposite direction (becuase d is above the fulcrum and the tangent is here horizontal) The spring-wire stands have bent until the stress on the wire is equal to the forces f and f-f' in each case.

There are three forces at work, apart from the spring in this wire. (the downward weight through c, the upward support through a, and the horizontal support through d.) Ti straighten it the only way consistent with known Maya

out the wire
/we can push point (a) to the right. Confining ourselves to Fig 13

what horizontal
we must find ~~what~~ force applied to point(a)will neutralize f - f'. ~~To~~

Now if one horizontal thrust is to be neutralized by another, purely
~~Of course~~ actually the weight has been helping to bend the wire support
vertical forces can hav nothing to do with the ~~m~~atter. And it is a
purely korizontal force we are after - side thrust.
~~and to eliminate this we~~ will The best way to visualize this will be to

shift our vault unit to a new~~s~~ spring wire stand, with an axis passing

this time through the center of gravity. We will assume that the spring

has the magical propo~~r~~ty of being able to support the weight of the

vault, but will neverth~~e~~less present no resistance whatver to bening

to one side or the other in response to horizontal forces. This eliminates
point a as a fulcrum, which~~n~~ is proper, since all it does is to furnish
vertical support to the vault.
The moedel will stay in any position we give it (as in

now
figures 1 and 2) since the axis/passes through the center of gravity/ and w
we are thus assured that the actual weight has no effect on the horizontal
forces we are investigating.
When we apply the force f of Fig. 12, in the same way, the support will

bend indefinitely, without changing the position of the vault with

reference to its upright position. Now by an act of imagination endow the

vault with the same rotational tendency it had in figure 13; that is,

tangental
a/force ~~wxxxcbx~~ equal to the effect of w x cb when applied at point d.

This of course, is the same as f-f' of Fig. 13, but in the opposite ~~direci~~

direction. It is perfectly obvious that now this force ~~is~~ will rotate the

the vault around the center of gravity (the new axis) not around point

a as in Fig. 13. ~~Forcexf-f'~~ To prevent rotation we must introduce we must make the effect of ~~khx~~ a horizontal force below the center of gravity equal to f-f'. If we choose to place this at the base, the lever on which ~~it~~ the horizontal force will act will be ~~ef~~ ch, and its effect equal to cg, whdih is ~~the-vault-height-less-~~ the height of the center of gravity above the vault spring. If is obvious that if, ~~xxxhxxxx~~ the center of gravity ~~ix~~ is at half the vault height, the force necesary at the base to prevent rotation will the equal to the f-f' applied at the top, stop the latter from moving out (to the left in Fig 14) will be ~~xqxxlxequalxtox~~ since the respective levers, ce and cb are equal. If

~~Instability results from factors:~~ two

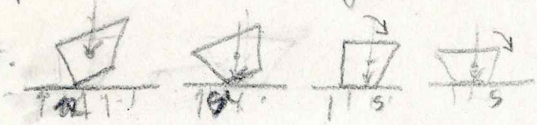
A successful vault in equilibrium – stable or unstable.

If unstable, it results from horizontal thrusts, ~~inward at the top~~ of the outer half-v. – in at top, out at bottom.

Both must be met.

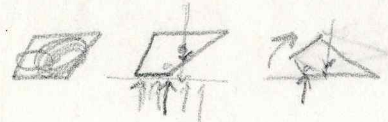
Before discuss ways of meeting them consider forces themselves.

Stable half vault.



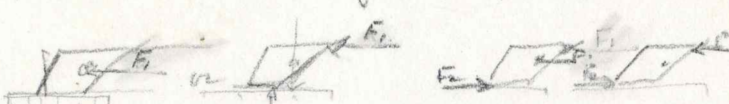
Def. of eqby.
- Center of gravity
- to attain eqby every force met by equal reaction.
- Simple + obvious but useful later.

Unstable ½ v – a 'couple' from force of (wei) + friction thy s. (see notes)



To ~~stabilize it~~ hold it erect: since in this form vertical forces cause a fall, can only prevent it with horizontal ones – can apply them in a Maya bldg only at the base and against the soffit slope or at its top. (by a vault seen) (by the roof cap)

:



~~Either case~~ model will slide across table (friction aside) unless an equal and opp force is applied on the other side. ~~For~~

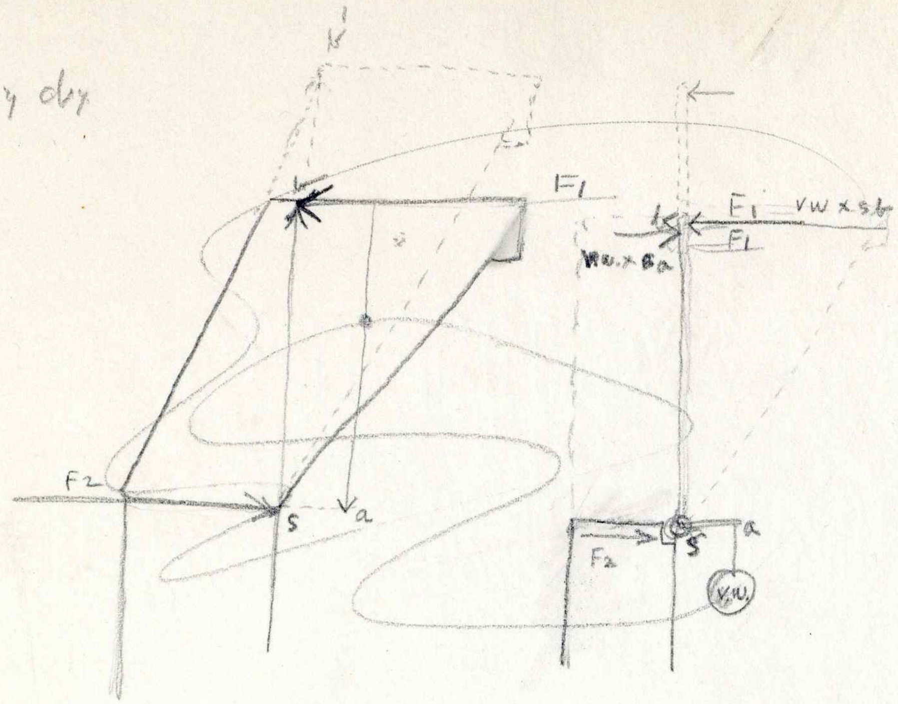
F_1 + F_2 equal in amount in all three cases.

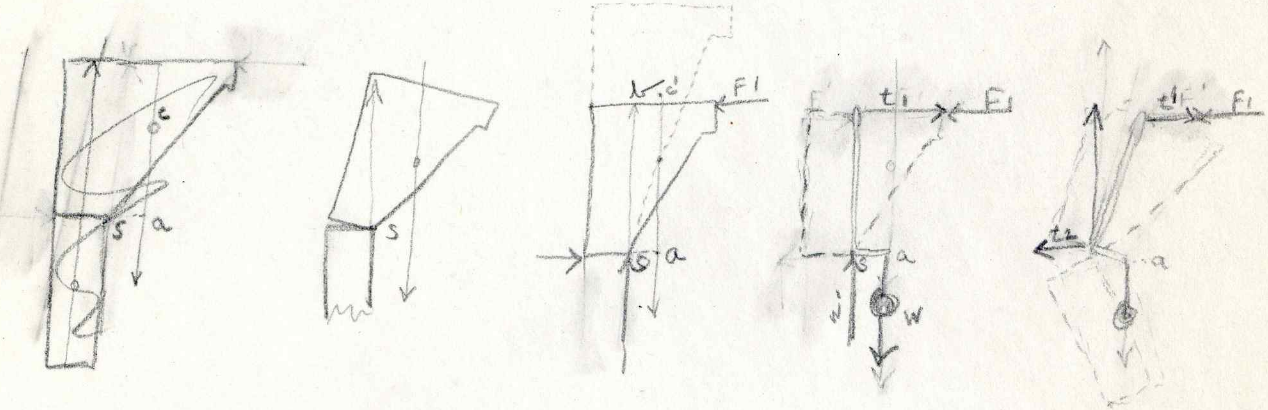
Def. Center of gravity + equilibrium above.

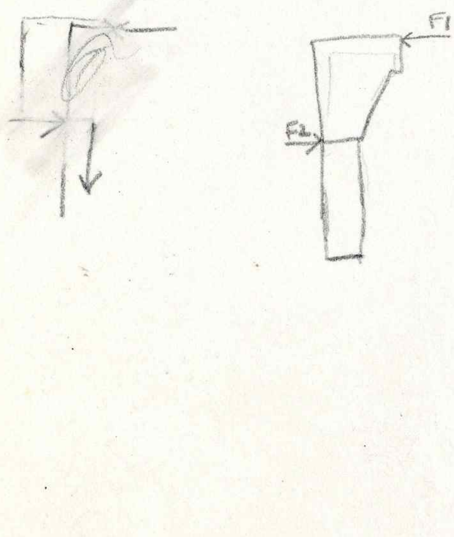
Amounts:

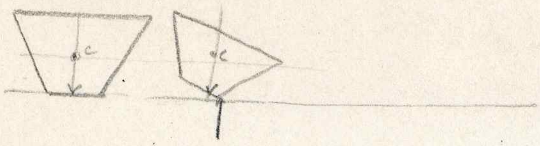
38.00
31.00
31.06

B.y dx








c
c

A If we arrange this in the same manner as we did a more (but more forceful)
stable vault in Figs. 6, 7, and 8, the same lamentable effect will be noticed on the wall. the difference is that it will not stand alone on the wall as did the other in Fig. 10, but it will fall off as in Fig 11. If we disregard the effects of mere inertia, the side-thrust ~~will~~ effect ~~~~ will not operate on the wall in Figs 10 and 11, because here there is nothing for the it to work against.

B at any point in the vault the rotational force (to be thus met) will of course ~~be~~ ~~~~ act on a tangent to the circle described by that point in the course of the rotation. therefore at b this turning force is purely horizontal (a tangent is at right angles to the radius ~~from~~ to the point of contact, and the radius a-b-d is vertical). the vault is a rigid solid and therefore our outside supporting force will pass through the center of gravity (c) (which tends to move a tangent directed downward) to point (b), were the force is purely horizontal. this is were it becomes effective in stopping the rotation. It must, obviously, be equal to the full turning force (weight times cb) since it has no lever to multiply its effect. Our wire support has bent to the left. ~~To straighten it~~ that is

Our wire support has bent to the left, but as soon as we we have ~~~~ prevented the falling of the vault, but as soon as we have done so we have introduced the side thrust again. Our horizontal pressure (equal to the torque effect at bc) has pushed the whole system to the left, besides righting the half-vault. Since the vault cannot turn about a, it tends to turn(a) about ~~~~ the point where it meets resistance (b).