

STEREO PHOTOGRAPHY  
FOR THE ARCHAEOLOGIST

An Extra Dimension for Records and Education

A Report from the Museum Applied Science Center for Archaeology

The University Museum  
University of Pennsylvania  
Philadelphia, Pennsylvania

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STEREO PHOTOGRAPHY FOR THE ARCHAEOLOGIST

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## STEREO PHOTOGRAPHY FOR THE ARCHAEOLOGIST

### PART I: THE PRACTICE OF STEREOSCOPIC PHOTOGRAPHY

#### The Purpose of this Report

Three-dimensional illustrations are an aid to teaching and communication. Stereo photography adds the dimension of depth. It will improve the viewer's comprehension of most scenes; furthermore, stereophotography is necessary to show the true three-dimensional structure of some scenes. This report outlines the techniques of stereo photography of excavations, statues, small finds, and architecture. The emphasis is entirely with the application of stereo photography for illustration and recording and not for mapping or plan making from photographs.

This report outlines the procedure of taking stereo photographs by sequential exposure with one camera: taking a picture of a scene, moving left or right a short distance, and taking a second picture of the same scene. The separation of these photo positions will sometimes be a greater distance than the separation of one's eyes. A stereoscopic view of the resulting pair of photos will give the effect of having an effective eye separation greater than that which is humanly possible; this will enhance depth perception.

#### The General Principles of Stereo Photography

Stereoscopic vision and photography are based on the principle that the appearance of three-dimensional scenes is different from separated viewpoints; that is, the foreground of a scene is displaced relative to its background when viewed from two points. This phenomenon is called parallax. Figure 1 illustrates the parallax one sees when looking down on the apex of a pyramid.

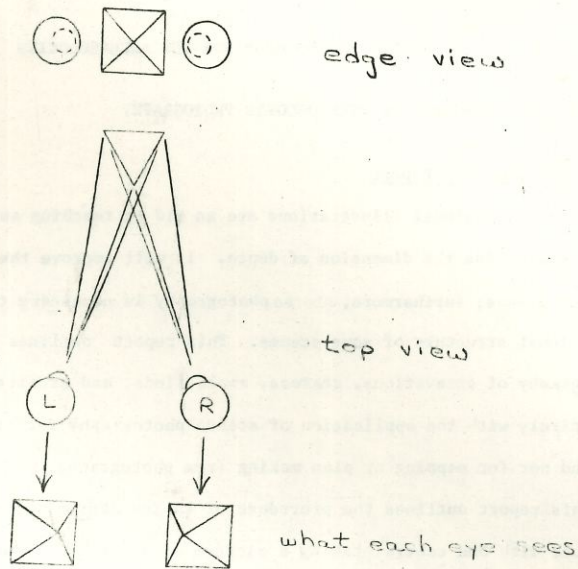


Figure 1: Binocular parallax of a pyramid

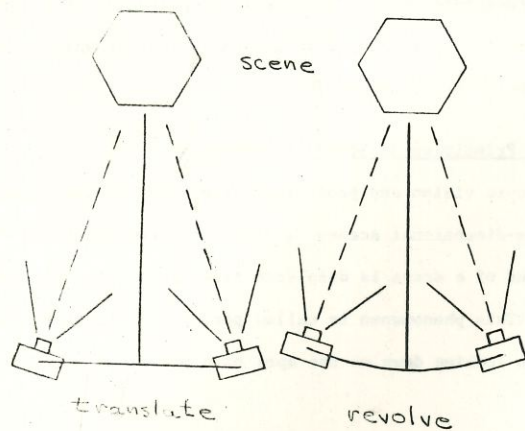


Figure 2: Converging alignments of camera positions

The two viewpoints which humans normally have are determined by an eye separation of about 65mm. With stereo photography, other viewpoint separations are possible. With a small separation the difference in the scene from the two viewpoints can be too small for three-dimensional depth perception. However, if the stereo separation is too large, the eyes and brain cannot combine the two dissimilar images and the scene is just an eye-straining confusion.

Many photographers who take three-dimensional pictures always use a photo point separation approximately equal to one's eye separation. Either they use commercially available stereo cameras or they use a general purpose camera and take a stereo pair by photographing a scene, shifting their weight from one foot to the other to move the camera slightly, and then photographing the scene again. A more valuable and vivid 3-D view requires an understanding of maximum photo point separation as it is determined by the dimensions of the scene and the parallax limits of easy stereo viewing.

Photo point separation is determined primarily by the depth of the scene and the distance to the scene. The depth of the scene is determined by the region of interest for 3-D viewing. It can be the depth of a statue or building as it is visible from the photo point; in landscape photography, it can be effectively infinite if the far horizon or clouds are important to the picture. Sometimes the foreground and background of the scene of interest do not need to be included in the determination of the depth of scene if they are out of focus or have a plain or unobtrusive texture, such as a bare floor or wall or a clear sky. Even if this is not the case, it is sometimes necessary to insure

adequate depth perception in the scene of interest by deliberately causing excessive parallax in the foreground or background. The distance to the scene is simply the distance from the photo point to the beginning of the scene of interest.

The procedure for stereo photography is as follows: frame the scene of interest in the camera and take the first photo, move left or right for scene parallax keeping the distance to the scene approximately constant, frame the scene of interest approximately as it was in the first photo by checking tilt and centering, take the second photo.

Photo point separation allows one to control scene parallax and therefore depth perception. In later sections of this report the method of determining photo separation for specific types of scenes will be explained. There is one general rule from which all these specific applications are derived: make sure that scene parallax is less than  $2^{\circ}$ . That is, when one moves sideways for the stereo pair make sure that the near point in the scene moves not more than  $2^{\circ}$  relative to the far point in the scene. For average scenes one will probably want to move so that the parallax is one half or one quarter of this value. If this maximum is exceeded it will become more difficult to see the depth of the scene when the photos are viewed because of their excessive stereo dissimilarity.

Scene parallax is easiest to determine when there is a sharp edge between the near and far points in the scene, which is often the case with architectural photography. A convenient angular measure is the width of the thumb on one's outstretched hand; this can be approximately  $2-3^{\circ}$ . To apply this "rule of thumb" when photographing a building, use a stereo separation such that a front corner of the building moves

relative to the background approximately half the width of the thumb as one moves sideways between photo positions. This rule is more difficult to apply to scenes that are rounded or that staircase with depth; fortunately, stereo separation is also less critical with these types of scenes.

One can easily measure the angular width of one's thumb by taping a meter stick to a wall at eye level and standing so that the tips of one's toes are 57 cm from the wall. The thumb on one's outstretched arm will cover a length of the meter stick; the apparent width of the thumb in centimeters is then equivalent to its angular width in degrees.

There is one general precaution one must take with this type of sequential stereo photography, and that is, there must be no movement within the scene during the time between taking the pair of photos. Watch out for moving people or wind-blown trees; also, if one waits more than several minutes between photos, there can be noticeable movement of shadows. One must also be sure that the stereo pairs are taken from approximately the same elevation. Never move obliquely up or down between photos unless the alignment of the camera frame is oblique also. While it is possible to correct for misalignment when the photos are printed or viewed, one will save film by starting correctly.

If moving objects are to be photographed, the shutters on two cameras must be synchronized to release at the same time. If the separation of the two cameras is greater than about two meters, it will probably be necessary to use separate tripods with electrical solenoids used to trigger both cameras simultaneously. With a separation of one meter or less, the two cameras can be mounted on a rigid bar, and both can be triggered with a paired cable release. Since precise synchronization of the two cameras will be difficult, objects which are moving very fast cannot be photographed.

#### How to Photograph Statues and Architecture

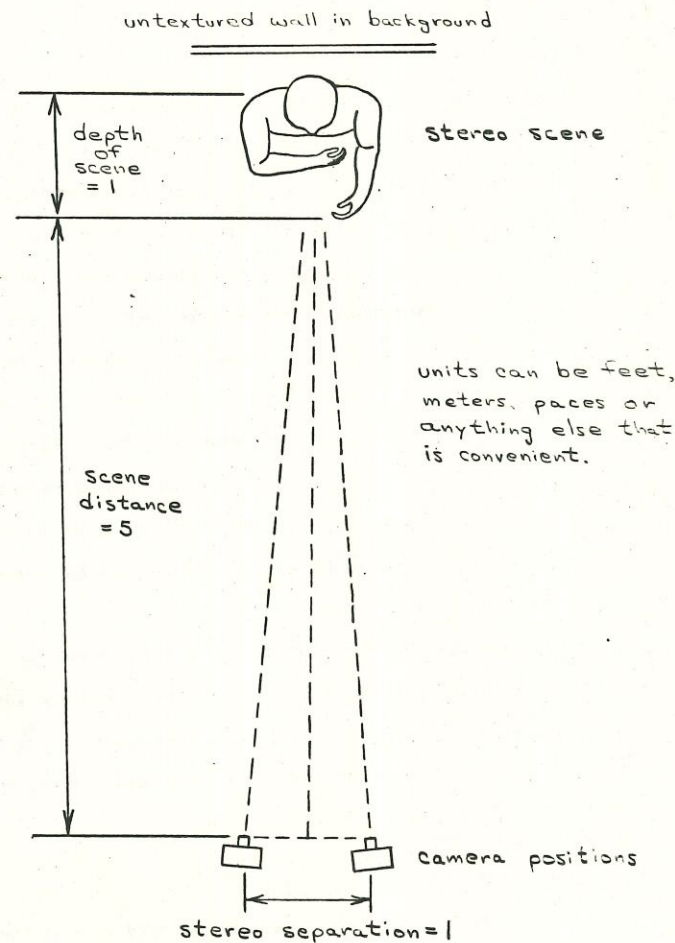
This type of stereo photography is about the easiest to do and the most fun to view. The subjects are usually medium relief scenes for which the distance to the scene is often in the range 2-20 times the depth of the scene. One can usually frame each of the pair of stereo photos the same and use a converging alignment of camera aim

directions as Figure 2 shows. However, for some architectural facades one must follow the technique outlined in the section on flat, low relief scenes in order to minimize horizontal perspective shift or keystoneing.

As an example of this type of photography, consider the case of a statue in front of a blank wall as shown in Figure 3. The depth of scene here is just the depth of the statue visible in the direction of view (if the wall had a strong pattern, the depth of scene would be the distance from it to the front of the statue). If the distance from the camera to the statue is five times the apparent depth of the statue, then the maximum camera separation is equal to the apparent depth of the statue. In most cases it would be best to use about half of this maximum stereo separation.

For other photo situations one can follow the rule that the ratio of stereo separation to scene depth varies roughly with the square of the ratio of the scene distance to depth as tabulated in Figure 3. Note that the unit of measurement can be anything convenient: feet, meters, or paces.

If distances are to be estimated from the position of the camera, remember that it is possible to estimate tangential distances more accurately than radial distances from one's viewpoint. Since these two measurements are related by an angle, it is possible to use the "rule of thumb" once more. In my case, if the thumb on my outstretched arm covers a strip of the scene which appears to be one meter wide, my distance to the scene is about 25 meters. The distance that one's thumb appears to shift across the scene when one looks at the scene with first one eye and then the other can be used also as a parallax range-



		quarter	quadruple		
ratio of maximum stereo separation to depth of scene	7%	25%	1	4	22
ratio of scene distance to depth of scene	1	2½	5	10	22
		halve	double		

Figure 3: An example of stereo separation,

finder. My proportionality for this measure is nine, so that in the above example my thumb would appear to jump a little less than three meters. If one stands a measured distance from a meter stick on a wall, one can measure his own rangefinder constants.

A graph for determining maximum stereo separation is given in Figure 4. Both distance to the scene and stereo separation are normalized to the depth of the scene in this graph. For example, if the distance to the scene is 8 times the depth of the scene, this graph indicates a maximum stereo separation of about  $2\frac{1}{2}$  times the depth of scene. Stereo photography with the pointing directions of the camera converging at angles greater than  $30^\circ$  should be avoided in general, for the convergence of the eyes required for viewing can be difficult. (Dalzell, 1936, p. 198).

The purpose of these procedures for determining stereo separation is simply to get one acquainted with proper stereo separation in several applications; with just a little experience one is able to estimate stereo separation well enough for most applications by simply observing the parallax of the scene.

#### How to Photograph Excavations

Landscape scenes are usually recorded with oblique photographs and therefore frequently have a depth of scene greater than the distance to the scene. If nearly vertical photos are taken from an elevated platform, follow the rules given in the preceding or following sections.

With oblique stereo photography, one's maximum stereo separation should be about 4% of the distance to the nearest point of the scene of

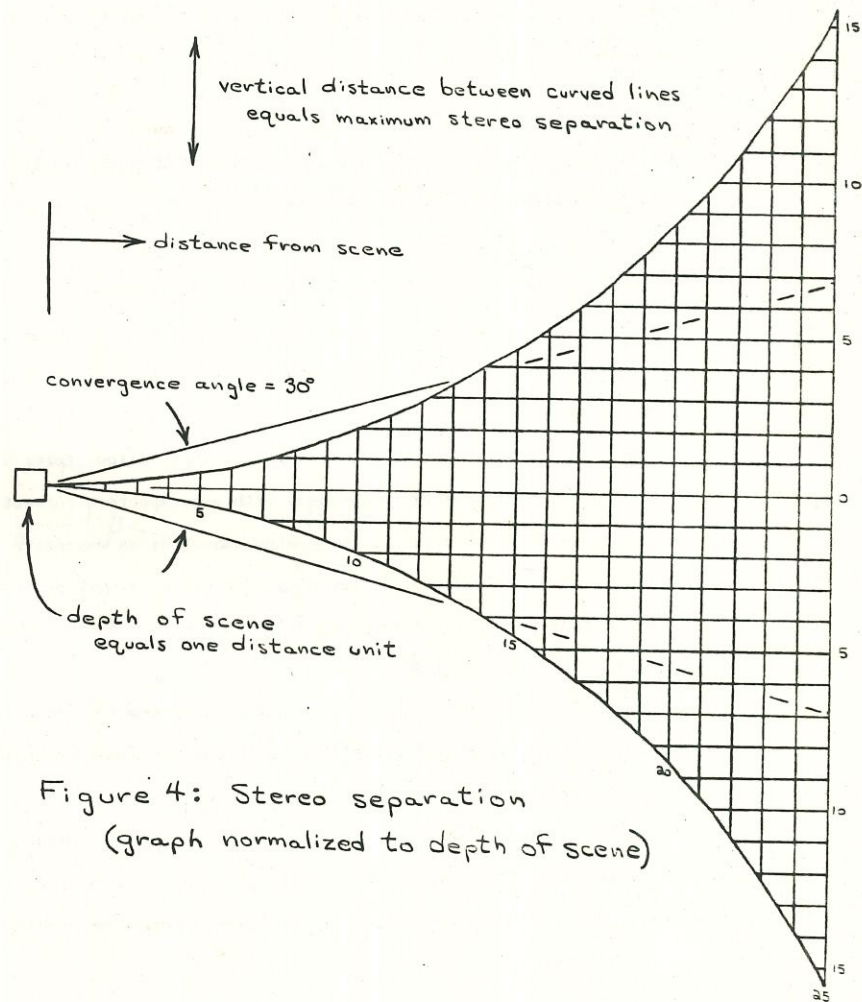


Figure 4: Stereo separation (graph normalized to depth of scene)

interest, as Figure 5 illustrates. In many cases a small camera separation may be used. If the bottom edge of the camera frame intersects the scene at  $1\frac{1}{2}$  meters from the camera, the maximum stereo separation is reduced to an interocular separation of 65 mm.

#### How to Photograph Flat, Low Relief Scenes

In this category bas-reliefs, inscriptions and coins are included. A large stereo separation is required to make the relief of these flat scenes visible. In the previous examples it has been possible to keep the camera centered on the scene for the pair of stereo photos. In this case excessive keystoneing, the distortion of squares into trapezoids, can result. In Figure 6 the difficulty with a rectangular scene is illustrated (Valyus, 1962, p. 174).

While a slight amount of keystoneing will still allow stereo viewing, keystoneing can be completely eliminated with a perspective correction camera. This requires either an expensive camera or an expensive special lens for a standard camera. Fortunately, one can do almost as good a job by keeping the camera aligned parallel to the flat scene instead of inclining it. Figure 7 illustrates this method.

The easiest way to do this with a 35 mm camera which has a lens with normal focal length is as follows: position the camera so that the scene fills the right hand two-thirds of the frame and the camera is looking perpendicular to the scene. Take a picture and then move right a maximum of 25% of the distance to the scene. Then place the scene in the left hand two-thirds of the frame. Again, make sure the

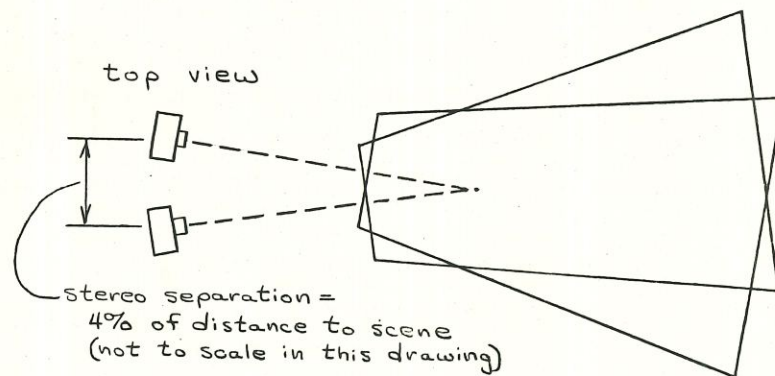
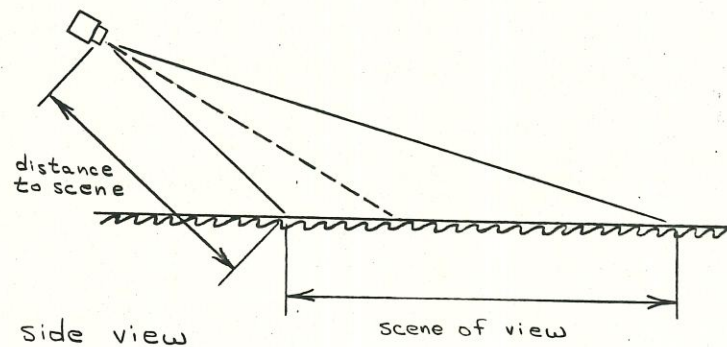
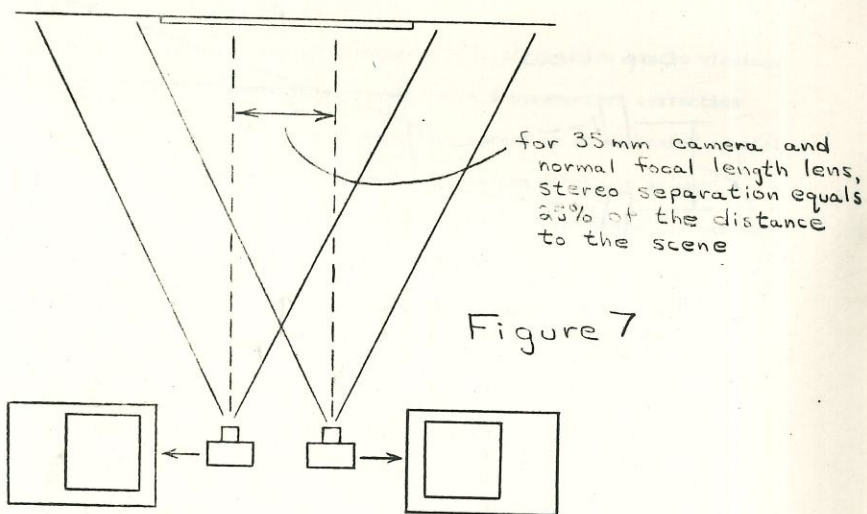
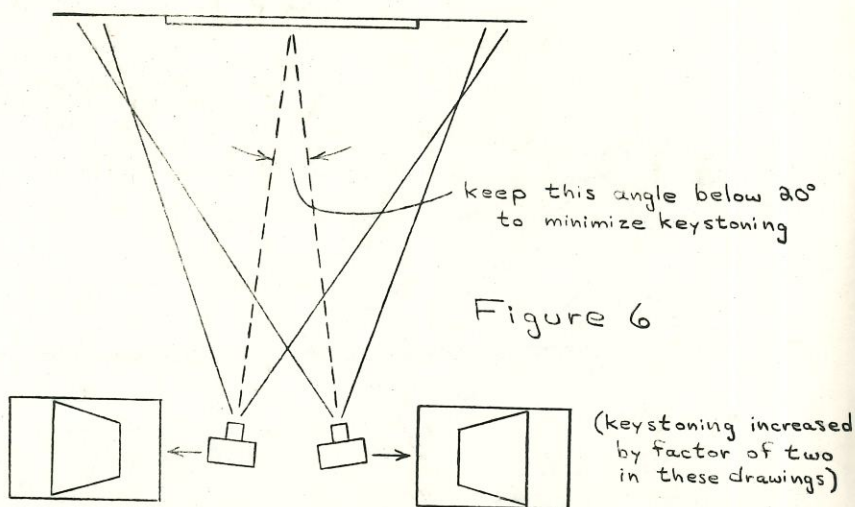


Figure 5: Oblique stereo photography



Photography of flat, low relief scenes

camera is perpendicular to the scene and take the second picture.

A portion of each frame will furnish a stereo view of the scene common to both photos.

The stereo separation for a square stereo overlap varies with the focal length of the lens as given in this table for 35 mm cameras.

Focal Length of Lens	Stereo Separation as a Percentage of the Distance to the Scene
24	50%
48	25%
60	20%
120	10%

There is one general rule which is independent of the lens. When looking at the scene through the camera's viewfinder, estimate the width in the scene of one third of the frame. This will be the stereo separation. It is possible to use less stereo overlap if one wishes greater depth enhancement, but one will also be wasting more of the available area of the film.

The stereo photography of coins or intaglios is easy with a single lens reflex camera. Select a lens for close-up work and mount the camera on a stand. Frame the object at one end of the viewfinder and take the first photo. Slide the object, without rotating it, so that it is in the other end of the viewfinder and take the second picture. The separation of these photo viewpoints will often be less than the spacing between one's eyes.

#### How to Photograph Small Objects

While small, low relief objects have been discussed in the previous section, other special techniques can be applied to items such as stone tools

and terra-cotta finds which can also be moved easily. The information on photographing statues given before can sometimes be applied to these objects.

Here again, the depth of scene is important. Note that for a cylindrical object with a flat-textured background the depth of scene is usually the radius of the cylinder, not the diameter.

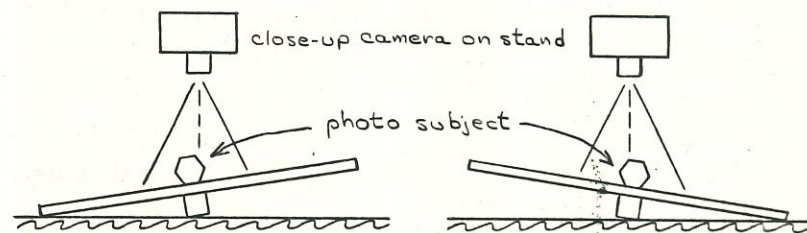
Since with small objects it is often easier to rotate the object than move the camera, the following table will help to estimate the best angle. Be careful with angles of rotation greater than about  $30^\circ$ ; since this can exceed the normal angle of convergence of one's eyes, there can be difficulty in viewing the photos.

Ratio of Scene Distance to Scene Depth	1	2	5	10	20
Maximum Angle of Rotation	$2^\circ$	$4^\circ$	$10^\circ$	$20^\circ$	$40^\circ$

If one rotates the object in front of the camera, a plain background must be used. In most cases the object must be illuminated by diffuse light (Kodak, 1969, p. 61) or the lights must rotate with the object in order to prevent confusing shadows from causing difficulty with stereo viewing (Symons, 1969, p. 1468).

One can take a sequence of photos around an object by putting it on a turntable which has a protractor for setting angles; later one can view adjacent pairs of photos.

A very easy way to rotate small objects is with a tilting table (Wienert, 1960). Figure 8 illustrates how this is done. A ridge under a flat board allows the object to be rotated easily through an angle determined by the design of the table. Since the angle of rotation is not critical, a single tilt-table



ratio of ridge height to total table width	0.87%	1.7%	3.5%	4.4%	6.6%	8.8%	13.4%	18.2%
rotation angle	$2^\circ$	$4^\circ$	$8^\circ$	$10^\circ$	$15^\circ$	$20^\circ$	$30^\circ$	$40^\circ$

For example, if a terra cotta fragment with 2 cm of relief is photographed from a distance of 20 cm, a rotation angle of about  $15^\circ$  would be suitable. A 20 cm square tilt table with a 1.3 cm square cross section rod under it could be used.

Figure 8: A tilt-table for small finds

can be used with specimens of somewhat different sizes. One can design a more elaborate table to vary the angle of rotation or to minimize the horizontal displacement of the object when it is tilted.

#### Viewing Stereo Photographs

Some people unfortunately do not have good enough stereo vision to enjoy viewing stereo photographs. About 2% of the population has no stereo vision at all and another 15% has only weak stereo vision (Julesz, 1971, p. 270; Howard, 1970, p. 143).

Many people, however, have very acute stereo vision and can perceive the separation in depth of an object at  $\frac{1}{2}$  kilometer against a far horizon (Rabben, 1960, p. 140; Hallert, 1960, p. 57). Stereoscopic acuity for both far and near objects can be enhanced by "spreading one's eyes" more than the average 65 mm spacing we have (Nowicki, 1952, p. 524; Howard, 1970, p. 143).

With photography we can easily select any amount of depth enhancement we wish. For example, if we take photos with a camera separation of ten times the separation of our eyes and then view the films, we will seem to be looking at a model of the real scene that is roughly ten times nearer to us and ten times smaller (Purves, 1969, p. 762; Hallert, 1960, p. 64) and our depth acuity is increased also by this factor. In aerial photography, the distance between photo points can exceed 2 km giving a depth enhancement of over 30,000 times (Rabben, 1969, p. 141; Hallert, 1960, p. 65).

Most people cannot look at stereo photos without optical aids. This is because of the difficulty we have in trying to keep each eye on separate photos; our eyes tend to converge to one photo (Gregory, 1970, p. 125; Avery, 1968, p. 25).

The simplest way to view stereo photos is with a lens stereoscope. This instrument consists of a pair of 2x to 3x magnifying lenses separated by the

distance between one's eyes. Make contact prints of black and white 35 mm stereo photos and place a pair under the lens stereoscope. Separate the photos so that the distance between corresponding image points in the pair is about 55 mm (Avery, 1968, p. 26; Nowicki, 1952, p. 531; Howard, 1970, p. 149); note that this is less than the separation of one's eyes. Be sure that the right hand photo that was taken is to be viewed by the right eye; otherwise one will see an impossibly inside-out confusion. With a little juggling of the viewer and photos one should be able to get the two photos to fuse into one 3-D scene.

Since the maximum usable width of a photo viewed with a lens stereoscope is 55 mm, 35 mm or 2 $\frac{1}{4}$  inch format cameras are excellent for stereo photography. Also, for example, note that a camera with a 4x5 inch or larger format would require that its photos be cropped or reduced in size during printing.

One can also make 2x to 3x enlargements of the negatives but one may have to crop them slightly to furnish the 55 mm separation, or one can view the enlargements by bending one of the photo edges temporarily upward so that the photos do not overlap when viewing near the center of the photos. Note that it can be impractical to dodge or burn-in while printing enlargements because of the difficulty of treating both halves of the stereo pair the same.

Correct for stereo misalignment by cropping enlargements and framing the stereo image relative to the edges of the photo (Kingslake, 1951, p. 192; McKay, 1948, p. 55; Kaiser, 1955, p. 76; Dalzell, 1936, p. 163). Stereo viewing gives one the impression of looking through a window at a three-dimensional scene. Except for some dramatic purposes, it is best to insure that the scene is entirely on the far side of the window resulting from the photo's framing. In order to do this, crop the left hand photo of the pair

to include the scene of interest; make it less than 55 mm wide but any desired height. Locate the nearest point to the observer in the photo and then crop the right hand photo so that this point is the same distance from the left and right edges in both photographs. This will put the nearest point in the photo right on the "window pane." To move the entire scene slightly behind the window crop the right hand photo a slight amount to the left of the position indicated above. This idea is illustrated in Figure 9.

One can make a stereo viewer for color slides by simply fastening together two inexpensive slide viewers. These usually have an adjustable focus on their lenses and diffusing screen behind the slide. Since it will probably be impossible to bring the two viewers close enough together so that the slide images are separated by 55 mm, one will have to duplicate a 100-year-old invention known as the Brewster Stereoscope (Purves, 1969, p. 1457). As Figure 10 illustrates, vary the spacing and angle of the viewers to take advantage of the prismatic effect of the lens to enable one to fuse visually the more widely separated photos.

Since one cannot make adjustments easily in the positions of the slides while they are in this viewer, it is important to frame the scenes accurately when they are photographed. However, for casual viewing, one's eyes will readily adjust to minor misalignments.

Most stereo viewers have lenses with focal lengths that are approximately equal to the focal length of a normal lens on a 35 mm camera, about 50 mm. When viewing contact prints or transparencies each of one's eyes will then see normal perspective (Kodak, 1967; Larmore, 1965, p. 179; Purves, 1969); this has been the basis of the rules given before on stereo separation. If a telephoto lens is used for stereo photography, determine the stereo separation by assuming that the actual distance to the scene is divided by the ratio

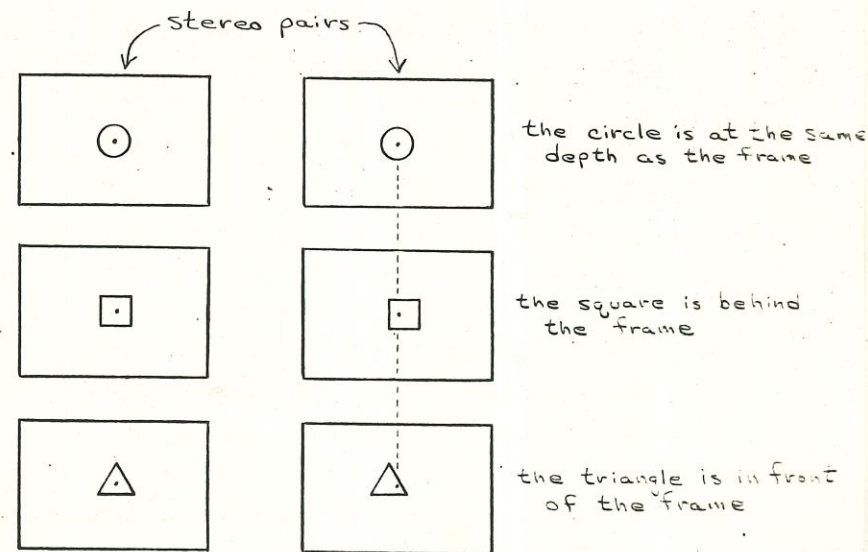


Figure 9: Framing stereo photographs

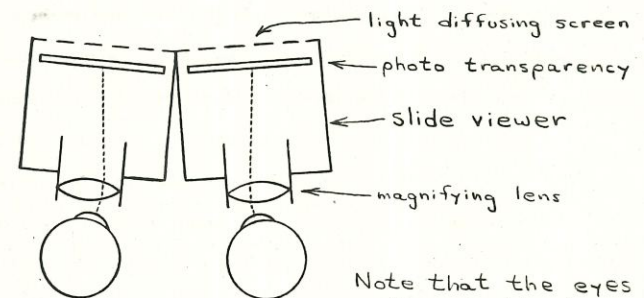


Figure 10:  
A stereo slide viewer

Note that the eyes look through the inner edge of each lens. Most slide viewers require this prismatic effect for convergence of the eyes.

of focal length of the telephoto lens to the focal length of a normal lens. In a related manner, it is possible to increase stereo separation if one uses a wide-angle lens.

If one plans to enlarge the photos before viewing them, remember that this is equivalent to using a telephoto lens. However, enlargements can often be viewed with correct perspective by using a mirror stereoscope (Nowicki, 1952, p. 528).

#### Publishing Stereo Photographs

A stereo photograph provides an excellent way of illustrating a site and artifacts more clearly in publications. For example, with some dark colored or shiny objects such as obsidian tools, it is very difficult to show three-dimensional shape in one photo. With color photography, objects must often be photographed in flat, diffuse lighting that eliminates the shadows which give good depth cues in a single photograph. Both of these cases can be helped by publishing stereo photographs.

While some archaeologists will have a lens stereoscope handy for viewing published photos, one must consider including viewing equipment with the report. Flat, Fresnel lens stereoscopes are made of inexpensive and flexible plastic and can be included with an article (Julesz, 1960). Cardboard and plastic viewers for classroom use or publication are available from Taylor-Merchant Corporation, 24 West 45th Street, New York City 10036. Because of the magnification of the lenses of these viewers, the half-tone dots on the printed photo may be visible; this is objectionable, but not serious. It is possible to publish color stereo photos with this method; however, the photos must be less than about 50 mm wide.

Most people find it easiest to view stereo photos as anaglyph prints. In this process the stereophotos are printed, one red and one green, directly on top of each other (Dudley, 1965, p. 96; Nowicki, 1952, p. 526; Hallert, 1960, p. 62; Valyus, 1962, p. 188) and viewed with a red celluloid over one eye and a green celluloid over the other. While the viewer will be quite inexpensive, a quality, two color print will add to the expense. When good color control is used for both the printing inks and the celluloid filters, the viewing is excellent (Julesz, 1968; Gregory, 1970; Fejes Toth, 1964; Kosofsky, 1970). Anaglyph viewers are available in large quantities from Brownie Manufacturing Company, 261 Broadway, New York City 10007. One may publish a photo of any size with this method, but it must be black and white originally.

A third method has, I believe, been too little used; this is the single mirror type of stereo viewer (Julesz, 1967; Purves, 1969, p. 1457). With this technique, one photo of the stereo pair is reversed left-to-right in printing and viewed in a mirror's reflection as shown in Figure 11. A normal glass mirror cannot be used, but an inexpensive plastic mirror which reflects only from its front surface could easily be included with the report. This technique can be used with a photo of any size and also with color prints.

#### Other Related Topics

Underwater Photography: Stereo photography in underwater archaeology has been applied primarily to mapping (Höhle, 1971; Bass, 1966; Bass, 1968; Green, 1971; Kodak, 1972). Stereo separation for photography of underwater objects is determined just as it is with terrestrial photography. Photography of underwater objects from above the surface of the water requires that

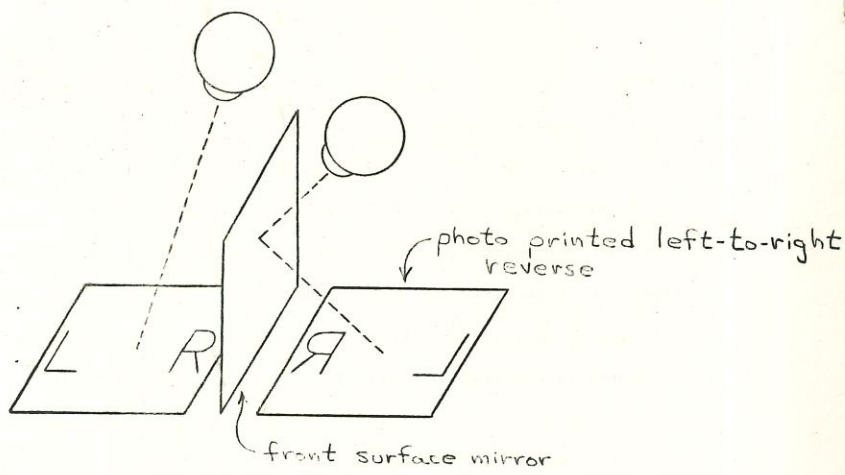


Figure 11: A single mirror stereo viewer

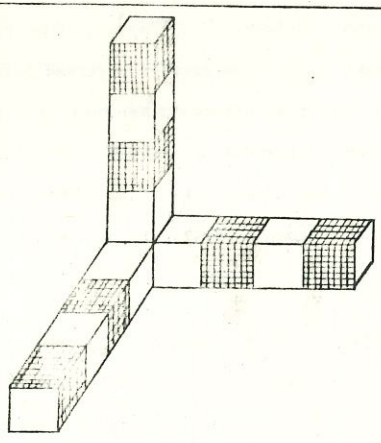


Figure 12: A 3-D photo scale

special camera filters be selected for greater visibility of deep objects (Colwell, 1960, p. 93; Helgeson, 1970; Mertens, 1970; Specht, 1973).

**Stereo Drawing:** Both contour maps and plans of sites can be presented as stereo pairs to clarify the dimension of depth in drawings (Gay, 1971; MacMahan, 1972; Gregory, 1970, p. 128, 168).

**Maps from Photos:** The preparation of accurate, three-dimensional maps from stereo photographs is a technical and detailed process (American Society of Photogrammetry, 1952; Hallert, 1960); it is easier, however, to make two-dimensional or plan maps. Both in architecture and archaeology, stereo photography has found application in the precise drawing of objects large and small (Carbannel, 1965; McFadgen, 1971; Whittlesey, 1966; Maruyasu, 1965; Simpson, 1967; Schwarz, 1964; Wiltshire, 1967; Shmutter, 1970; Karara, 1972; American Society of Photogrammetry, 1971; Graham, 1972).

For this type of photography it is a good idea to include a three dimensional metric stick in the scene, with perpendicular rods as shown in Figure 12. For use in measurement, align the rods with the principal dimensions of the object of interest. With fairly good accuracy, measurements can be taken from single photos by having such a metric scale in your scenes (Janke, 1972; Williams, 1969; Williams, 1972).

**Substitutes for Stereo Photography:** There is a simple optical method of seeing and photographing contour lines of constant depth on any object. A light illuminates the object through a grating of parallel lines; at the same time the object is viewed through the grating. The correspondence of the shadow of the grating on the object with the image of the grating

forms a precise topographic map (Takasaki, 1970; Takasaki, 1973; Wasowski, 1970). This must be seen to be believed, but it can save considerable time in contouring, determining volumes, and recording subtle shapes.

Another technique which one should keep in mind is slit photography. With this technique a cylindrical object such as a vase is rotated in front of a special camera similar to a panoramic camera. The film is advanced in synchronism with the object's rotation and a thin line of the cylinder is scanned along the film. This allows the entire circumferential view of an object to be seen in one "undistorted" photograph. This camera, the R.E. Periphery Camera, is made by Research Engineers, Ltd., Orsman Road, London.

Holography, the recording of three-dimensional shapes with laser light, may eventually prove practical, although it is not yet (Smith, 1969, p. 23; Hammond, 1973).

Projecting stereo photos: Stereoscopic slides can be projected on screens by either the anaglyph method mentioned before or by using polarized light. Both methods require that each viewer wear special goggles (Kingslake, 1951, p. 197; Rule, 1941a; Rule, 1941b; Spottiswoode, 1953; Dewhurst, 1954).

Aerial Photography: Stereo photography is invaluable for aerial reconnaissance and mapping. Three large manuals from the American Society of Photogrammetry provide good introductions to this field (American Society of Photogrammetry, 1952, 1960; Smith, 1968). The history of aerial photography applied to archaeology is described in a book by Deuel (1969).

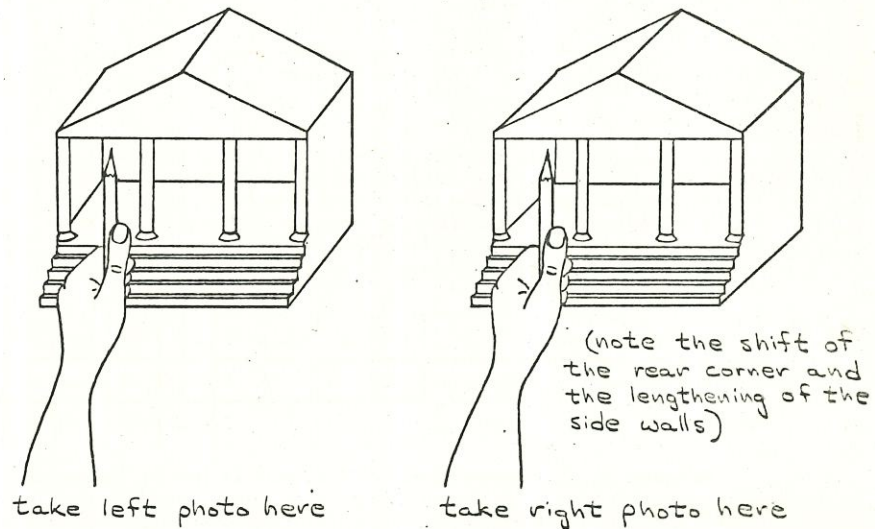


Figure 1: The pencil as a stereo measure

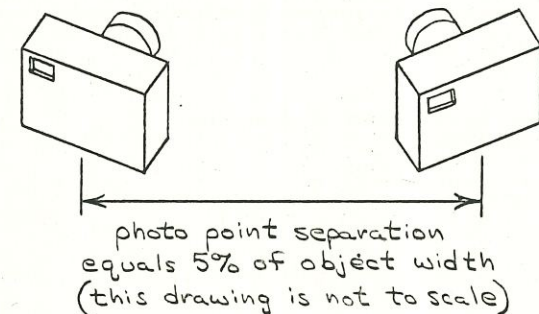
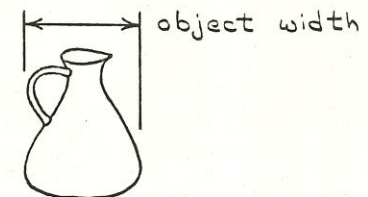


Figure 2: Stereo photography of objects

For landscapes, the distance between your photo points should be about 2% of the width of the scene at the nearest part of the photo. You can estimate this from the width of the scene that you see at the bottom of the camera's viewfinder; an accuracy to within 50% is again all that is needed. Figure 3 illustrates the procedure.

The following table can be used as an aid in the calculation of the stereo separation:

Width of the scene		100	200	300	400	500	600	700	800	900
Distance between stereo photo points	Landscape	2	4	6	8	10	12	14	16	18
	Object	5	10	15	20	25	30	35	40	45

For example, a landscape scene 400 feet wide as seen at the bottom of the camera's viewfinder should have a photo point spacing of about 8 feet; also, for a statue 65cm wide, a 3cm stereo separation is indicated, but this should be increased to the minimum 6cm.

After film processing, you can view your pair of stereo photo prints by placing them under a lens viewer as illustrated in Figure 4. Arrange the prints in the same left-right order as they were photographed. The distance between matching points on the pair of photos should be about 55mm; therefore, the width of the photos can also be no greater than 55mm.

By making minor adjustments in the positions of the photos and the stereoscope, you can align the two photos relative to your pair of eyes and thereby eliminate image doubling when you look through the twin magnifying lenses. You can verify this by closing one eye and then the other; the image should not jump. Try to view a sharp, contrasty part of the scene first. While it may take a minute before your eyes adjust to seeing stereo this way, the startling effect of the depth that you can see will be worth the effort.

Stereo photos which are color or black and white transparencies can also be viewed. Hold two small slide viewers together to create a makeshift lens stereoscope; alternatively, the strips of film can be mounted for viewing in commercially-available stereo slide viewers.

For a more detailed discussion on stereo photography, see the companion report to this one, "Stereo Photography for the Archaeologist", dated 15 May 1973.

Museum Applied Science Center for Archaeology  
The University Museum  
University of Pennsylvania  
Philadelphia, Pennsylvania  
26 October 1973

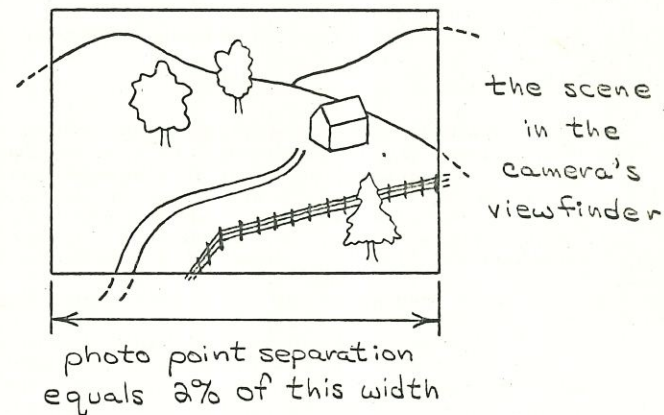


Figure 3: Stereo photography of landscapes

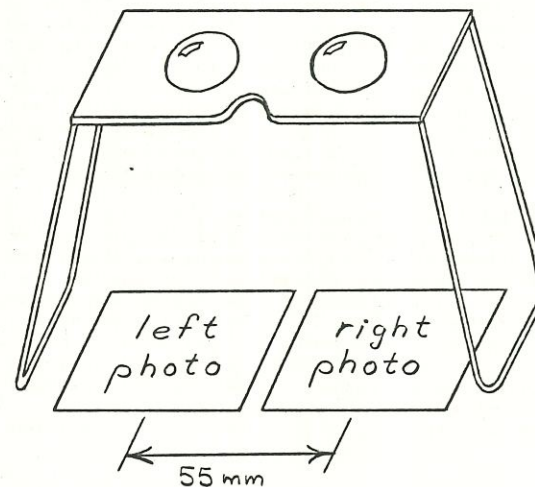


Figure 4: A lens viewer for stereo photos

## AN INTRODUCTION TO STEREO PHOTOGRAPHY

Bruce Bevan

Stereoscopic photography will add the dimension of depth to your illustrations. This report describes the basic technique of photographing landscapes, buildings and smaller objects in three dimensions.

For stationary objects, a single, standard camera is all that you need to reconstruct a three-dimensional view with stereo photography. Take two pictures of the scene you want to record, each from a different viewpoint:

- 1: Center the scene in the camera's viewfinder
- 2: Photograph the scene
- 3: Move left or right a short distance
- 4: Again, center the scene in the viewfinder
- 5: Photograph the scene again

The selection of the distance between the two points at which the pair of stereo photos are taken is described next. When you photograph distant scenes, the separation of these photo points will be greater than the distance between your pair of eyes; then, when you view the pair of photos, your perception of the depth of the scene will be enhanced.

In the scene which you will photograph, look for a point where the foreground and background of the scene appear to be near each other from your point of view; a tree branch against the horizon or a wall almost perpendicular to the picture are two examples. As you move sideways, this foreground point will shift sideways relative to the background.

Hold a pencil in your hand and stretch your arm straight out as shown in Figure 1. Select the photo points for your stereo pair such that the shift of the foreground relative to the background that you see with one eye closed is about equal to the "width" of the pencil.

Many scenes have no abrupt step between their near and far points. Fortunately, the accuracy of stereo separation is even less important for these scenes, and you can usually estimate closely enough. Also, you can use the length of the pencil to compare the shift between vertically separated foreground and background points in these more rounded scenes.

This technique is accurate and easy to use, but is not applicable if your camera has a wide angle or telephoto lens. The following methods will also let you approximate stereo separation; with them, you may use any type of camera or lens that you have.

For objects such as buildings and statues, the separation of the photo points should be about 5% of the width of the object; see Figure 2. However if only part of a large object is being photographed, measure only the width that will be in the picture. Except for close-up photography, when the object is less than 25cm from the camera, make the stereo separation 6cm or greater. An accuracy of 50% in the determination of the separation of photo points is all that is required.

Commercially available stereo photography equipment: There are several models of stereo cameras on the market now; most of these use a single camera equipped with two synchronized lenses separated by about 65 mm (Morgan, 1954; Kaiser, 1955). Stereo attachments are also available that can be put in front of the lens of a standard camera in order to split the normal frame into two stereo views. One good source of new and used stereo equipment is Olden Camera, 1265 Broadway, New York City 10001.

## PART II: THE BASIS OF STEREOSCOPIC PHOTOGRAPHY

### Cues for Depth Perception

Our perception of depth and distance is derived from a variety of visual information, called cues. While some of these cues require binocular vision, many are monocular. A single cue can sometimes give erroneous information (Ittelson, 1952), but a combination of cues increases reliability. Some of the major perceptual cues are listed below (Ogle, 1962, p. 251; Graham, 1966, p. 504). Some of them help to determine the absolute distance to an object while others indicate only the relative depth between objects.

**Relative Size:** The angular size of an object decreases in proportion to its distance, so that if we know the dimensions of an object, we can estimate its absolute distance from us. Linear perspective as typified by railroad tracks converging in the distance, is an example of this distance cue.

**Interposition:** Near objects can partially block the view of objects farther away. This cue does not allow one to measure either relative or absolute distance.

**Aerial Perspective:** The contrast of distant objects is decreased because the intervening air or water is not perfectly transparent. This cue is related to interposition and most commonly results from haze in the air.

**Light and Shade:** Highlight and shadow patterns reveal the relief of an object. This can be the primary depth cue in photographs of stone and terra-cotta figurines.

**Image Blurring:** If accommodation, the focus of the eye, allows one object to be seen sharply, another nearer or farther can be blurred.

In the language of photography, this is called the circle of confusion resulting when a point is not imaged at the camera's focal plane.

**Motion Parallax:** When one's head moves sideways, near objects appear to move relative to farther objects. While all of the above depth cues can be recorded on a single ordinary photograph, this one cannot except through the special technique of holography.

**Proprioception:** The muscles of the eyes which control accommodation and binocular convergence send signals to the brain which indicate object distance. This cue is surprisingly weak and unreliable for both absolute and relative distance estimation.

**Stereopsis:** The retinal disparity between one's pair of eyes is the best visual cue to relative depth. This cue to relative distance, which is a result of binocular parallax in a scene, tells one nothing about the absolute distance to an object.

### Binocular Disparity

Because of their separation of about 65 mm, each of our eyes sees a slightly different view of the same three-dimensional scene. The eyes converge to give the closest correspondence between this pair of images and then the brain interprets the remaining disparity as relative depth in the scene. Note that the word "parallax" usually describes the difference in a scene from two viewpoints while the word "disparity" usually describes the difference in the retinal image between one's pair of eyes; however, in most applications these words will be synonymous.

The measure of disparity is the difference in angular separation between corresponding points as seen from two separate locations; see

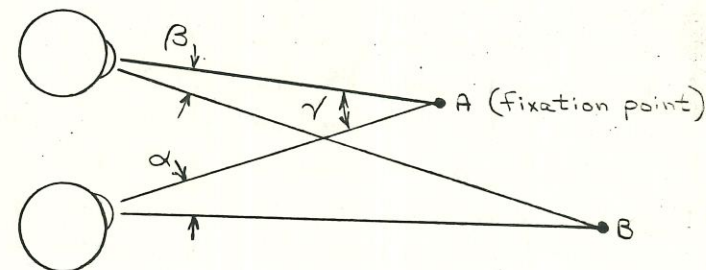
Figure 13. This disparity must exceed a minimum threshold to be perceived. That is to say, one has no stereoscopic depth perception of small objects at great distances. The difference between binocular parallax and the angle of binocular convergence is also illustrated in Figure 13.

Angular disparity changes with: distance to the scene, depth of the scene, and effective eye separation. With photography one controls effective eye separation with the spacing of the viewpoints of the photo; this is the stereo separation.

When retinal disparity is somewhat greater than the minimum threshold of stereoscopic perception, the brain's fusion of the binocular images causes them to be perceived as one three-dimensional scene (Pettigrew, 1972). As this disparity increases further, a maximum is reached and the two binocular images are too different to be fused. This phenomenon is called diplopia when images are doubled. It is easily experienced by looking at a distant object with a finger held up in front of one's eyes.

With a slight amount of diplopia good stereo vision is still easy and in fact so commonly experienced that it goes unnoticed. Further increase in diplopia allows one to estimate only roughly the relative proximity of targets. Even further increase yields only a confusion with no information about depth.

All of these limits to stereopsis increase with angle from the fixation point as indicated in Figure 14 (Ogle, 1962, p. 283; Le Grand, 1967, p. 217). Since this graph is for a typical observer, the limits can vary considerably (Ogle, 1950, p. 65).



$$\text{angular disparity between points A \& B} = |\alpha - \beta|$$

$$\text{binocular convergence} = \gamma$$

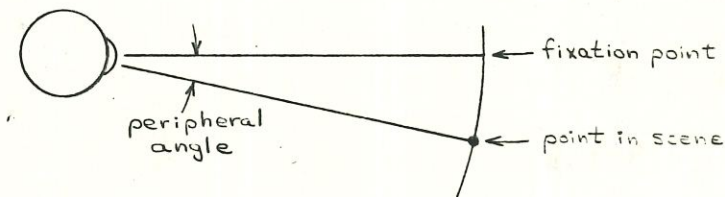
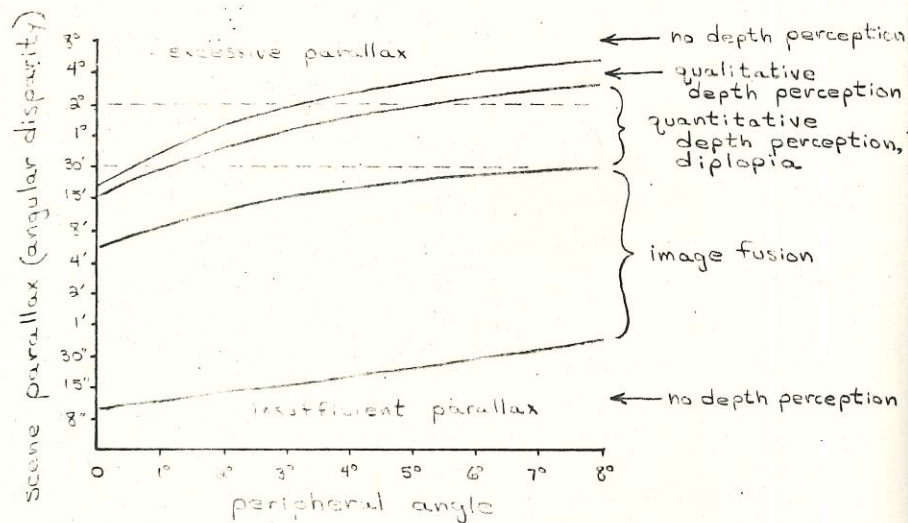


Figure 13: Some ocular angles



data from: (Ogik, 1962, p. 283) & (LeGrand, 1967, p. 217)

Figure 14: The limits of stereoscopic vision

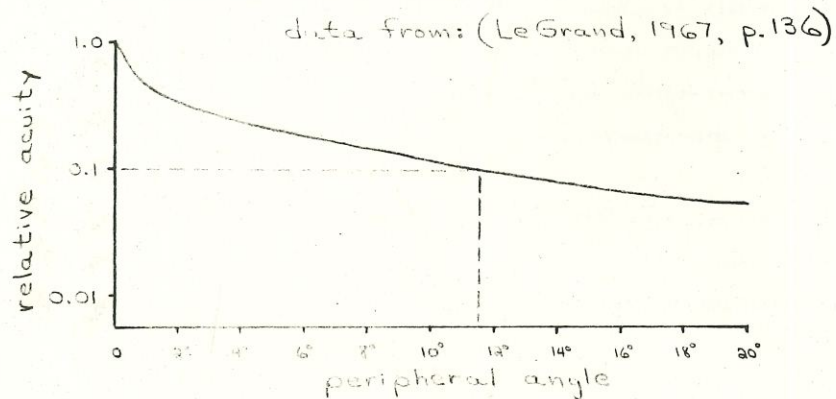


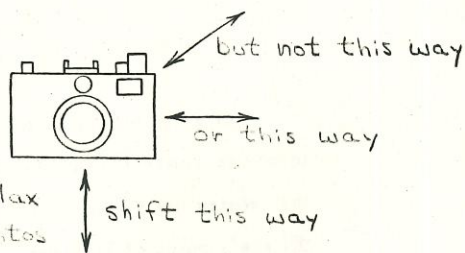
Figure 15: Visual Acuity

This graph should be interpreted in conjunction with the variation in visual acuity (Le Grand, 1967, p. 136) as shown in Figure 15. Note that acuity has decreased by a factor of ten at an angle of  $10^\circ$  from the eye's point of fixation. This indicates that one uses only the central portion of one's retina for accurate perception and the eye must get its detailed information by scanning a scene.

This figure suggests also that the depth of scene need only be considered as the largest depth found in any area within a  $20^\circ$  angle. For example, in a stairstepping or sloping scene greater than  $20^\circ$  wide, the effective visual depth of scene for the calculations of stereo separation can be less than the total depth of scene.

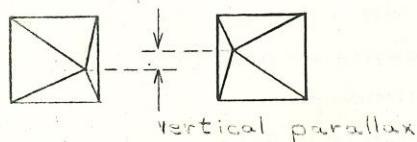
These two figures indicate that the  $2^\circ$  parallax chosen for stereo photography is a maximum practical limit. However, one may choose a greater or lesser parallax for specific applications. Earlier work by French (Judge, 1935, p. 28) indicated a maximum parallax of about  $0.5^\circ$  for stereo vision. If one wishes the best stereo representation, one will have to do some experiments to insure that one finds the optimum point where depth is apparent in the scene's areas of low relief and that excessive diplopia does not occur in the areas of high relief.

The above discussion has been on horizontal parallax between stereo photos. Incorrect alignment during photography or viewing can also add vertical parallax. This can be prevented by translating between stereo photo points parallel to one of the two directions of the camera's frame. Vertical parallax can also be eliminated later in printing or viewing, but this is difficult to do with color transparencies. Figure 16 illustrates the problem; not only does this vertical parallax not add

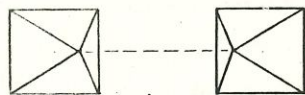


To prevent vertical parallax when taking stereo photos

If you translate obliquely between photo positions, you will not get stereo photos like this:



when you should have gotten this:



This vertical parallax can be corrected by rotating each photo before viewing

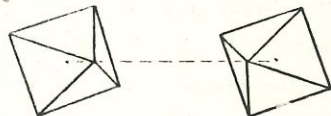


Figure 16: Vertical parallax

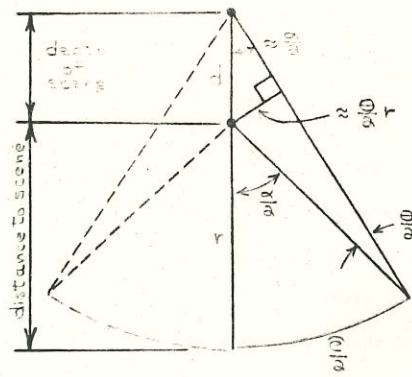
depth information, if it exceeds  $1^{\circ}$ - $2^{\circ}$  (Ogle, 1950, p. 66; Hallert, 1960, p. 55), stereopsis is impossible. Small amounts of vertical parallax can be corrected by the viewer's eyes with cyclorotation: each eye rotates about its axis of view.

When taking oblique stereo photos with a large angular separation, vertical parallax is prevented by aligning the camera in its two positions along a longitudinal line of an imaginary hemisphere centered on the scene. With a small stereo separation, camera alignment with a line of latitude can be used; this is usually easier to do and the resulting vertical parallax will be slight.

#### Geometric Calculations for Stereo Separation

There are several slightly different approaches to the determination of how the angular parallax of a scene varies with the separation of the viewpoints. Figure 17 illustrates the simplest analysis. Two points represent the near and far points in the scene. If one moves symmetrically around the scene along an arc centered on the near point in the scene, the far point appears to move through an angle called the angular parallax of the stereo separation. Because of the symmetry of this model, half of it indicates the geometry for this calculation of parallax.

A more accurate analysis is given in Figure 18. Two related derivations are given here: one assumes that the foreground is more important and keeps the near point in the scene centered by pivoting about it; the other assumes that the background is more important and has the far point in the scene as a pivot. Note that both approaches can be simplified to the approximation of Figure 17 for shallow scenes.



$\alpha$  = pivot angle about near point in scene  
 $\Theta$  = arc length of stereo separation  
 $\theta$  = angular parallax of scene

Assume  $\alpha$  and  $\Theta$  small so that  $\sin \frac{\alpha}{2} \approx \frac{\alpha}{2}$  &  $\sin \frac{\Theta}{2} \approx \frac{\Theta}{2}$   
 In radians,  $\frac{\Theta}{r} \approx \frac{\Theta}{d}$  and  $\frac{\Theta}{r} \approx \frac{\Theta}{d}$ ; therefore  $\frac{\Theta}{d} = \frac{\Theta}{r}$   
 and  $\frac{\Theta}{r} = \theta \frac{d}{r}$  or  $\frac{d}{r} = \theta \left(\frac{r}{d}\right)^2$

If we want the parallax to be at most  $2^\circ$ , then  
 $\theta \leq 2^\circ = \frac{\pi}{90} \approx 0.04$  radian

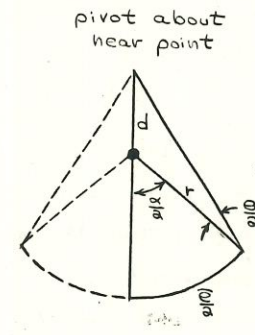
So that the stereo separation formula is:  
 $\frac{\Theta}{r} \leq 4\% \frac{r}{d}$  or  $\frac{d}{r} \leq 4\% \left(\frac{r}{d}\right)^2$

Several values from these formulas are:

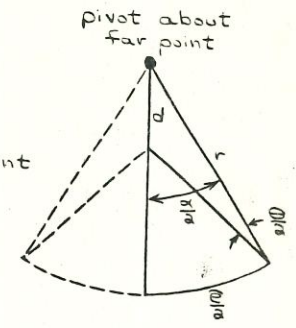
$\frac{r}{d} = \frac{\text{distance to scene}}{\text{depth of scene}}$	1	$2\frac{1}{2}$	5	10	25
$\frac{\Theta}{r} = \frac{\text{stereo separation}}{\text{distance to scene}}$	4%	10%	20%	40%	1
$\frac{\Theta}{d} = \frac{\text{stereo separation}}{\text{depth of scene}}$	4%	25%	1	4	25
$\alpha$ = pivot angle	$2^\circ$	$6^\circ$	$11^\circ$	$22^\circ$	$57^\circ$

maximum values

Figure 17: Approximate analysis of stereo parallax



$\alpha$  = pivot angle  
 $d$  = depth of scene  
 $r$  = distance to pivot point  
 $\Theta$  = pivot arc length  
 $\theta$  = angular parallax



$$\frac{r \cos \frac{\alpha}{2} + d}{r \sin \frac{\alpha}{2}} = \cot \left( \frac{\alpha}{2} - \frac{\Theta}{2} \right)$$

$$\frac{d}{r} = \sin \frac{\alpha}{2} \cot \left( \frac{\alpha}{2} - \frac{\Theta}{2} \right) - \cos \frac{\alpha}{2}$$

$$\therefore \alpha \approx \Theta \left( \frac{r}{d} + 1 \right)$$

$$\frac{r \cos \frac{\alpha}{2} - d}{r \sin \frac{\alpha}{2}} = \cot \left( \frac{\alpha}{2} + \frac{\Theta}{2} \right)$$

$$\frac{d}{r} = \cos \frac{\alpha}{2} - \sin \frac{\alpha}{2} \cot \left( \frac{\alpha}{2} + \frac{\Theta}{2} \right)$$

$$\therefore \alpha \approx \Theta \left( \frac{r}{d} - 1 \right)$$

These last approximations give a maximum error of 3% at  $\alpha = 20^\circ$

These equations are tabulated below for  $\Theta = 2^\circ$

$\alpha$ , pivot angle	$3^\circ$	$4^\circ$	$6^\circ$	$10^\circ$	$15^\circ$	$20^\circ$	$30^\circ$	$40^\circ$
minimum r/d for pivot about near point in scene	0.5	1.0	2.0	4.0	6.5	8.9	13.9	18.5
minimum r/d for pivot about far point in scene	2.5	3.0	4.0	6.0	8.5	11.0	15.9	23.4

Figure 18: Pivoting about a stereo scene

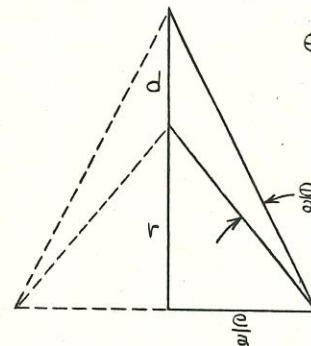
As one's distance from the scene to be photographed increases, it becomes easier to use a straight line separation between photo points. The analysis of the geometry of this problem is given in Figure 19. Once again, this exact analysis will yield a simple approximation in many cases. The exact analysis is tabulated in Figure 20 and plotted in Figure 4.

Several authors have determined stereo photo separations for specific types of scenes. (Symons, 1969, p. 1463; Wienert, 1960; McKay, 1948, p. 170; Kaiser, 1955, p. 117). Another author has developed a general formula similar to those above and has used an angular parallax of  $1^\circ$  (Judge, 1935, p. 38; Dalzell, 1936, p. 174). The similarity between some formulas for parallax is shown in Figure 21.

If one takes stereo photos of scenes with low relief by rotating the optic axis of the camera about the center of the scenes, it will be found that large angular rotations will cause excessive keystoneing. In Figure 22 the maximum swing angle is determined. As an example, a camera with a 50 mm lens and 35 mm film (24 x 36 mm format) should never be rotated over  $22^\circ$  in front of a scene with flat surfaces.

One can prevent keystone parallax in stereo photography by keeping the optic axis of the camera at the two photo positions parallel to each other. With 35 mm cameras it is convenient to move between stereo photos to give a two-thirds overlap in the photos; then a 24 mm square will be common to each frame in the stereo pair. While it is easy to determine the required stereo separation from an estimation of one third of the width of the scene in the camera's viewfinder, one can also determine the separation from the distance to the scene as shown in Figure 23.

All of the previous examples of parallax as a function of stereo separation in Figure 17 through 21 have assumed that the scenes had sharp edges. As



$\Theta$  = angular parallax, the angle of relative displacement of the scene's near point relative to its far point.

$d$  = depth of scene  
 $r$  = distance to near point of scene  
 $e$  = stereo separation

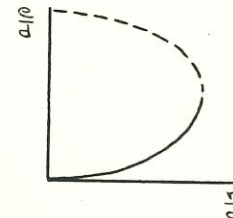
$$\frac{\Theta}{a} = \tan^{-1} \frac{r+d}{e/a} - \tan^{-1} \frac{r}{e/a}$$

$$\tan \frac{\Theta}{a} = \frac{(e/a)d}{(e/a)^2 + r(r+d)}$$

$$e = \frac{1}{\tan \frac{\Theta}{a}} \left[ d \pm \sqrt{d^2 - 4r(r+d) \tan^2 \left( \frac{\Theta}{a} \right)} \right]$$

$$\frac{d}{a} = \frac{1 - \sqrt{1 - 4 \frac{r}{a} \left(1 + \frac{r}{a}\right) \tan^2 \frac{\Theta}{a}}}{\tan \frac{\Theta}{a}}$$

$$\frac{r}{a} = \frac{\frac{d}{a} - \sqrt{\left(\frac{d}{a}\right)^2 - 4 \left(1 + \frac{d}{a}\right) \tan^2 \frac{\Theta}{a}}}{\tan \frac{\Theta}{a}}$$



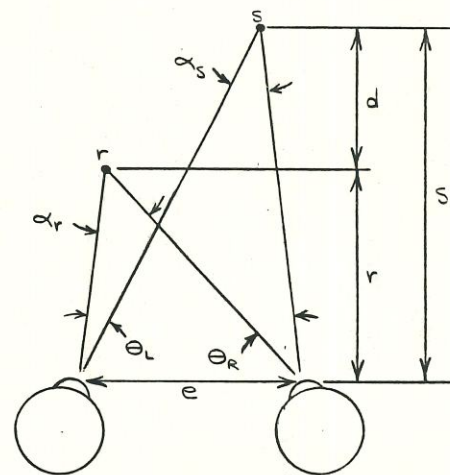
for  $r \gg d$  this simplifies to  $\frac{\Theta}{a} = \Theta \frac{r}{a}$   
 for  $r \ll d$  this simplifies to  $\frac{\Theta}{a} = \Theta$

Figure 19: The parallax of stereo translation

$\frac{r}{d}$ distance to scene depth of scene	$\frac{e}{2d}$ maximum stereo separation twice depth of scene
--	---

0.1	0.00192
1	0.0349
2	0.105
3	0.211
4	0.352
5	0.529
6	0.743
7	0.995
8	1.286
9	1.617
10	1.989
11	2.405
12	2.867
13	3.376
14	3.936
15	4.551
16	5.224
17	5.962
18	6.770
19	7.656
20	8.632
21	9.710
22	10.91
23	12.26
24	13.79
25	15.54

Figure 20: Maximum stereo separation



$$\alpha_s \approx \frac{\theta}{s}, \alpha_r \approx \frac{\theta}{r}$$

$$\alpha_r - \alpha_s = \theta \left( \frac{1}{r} - \frac{1}{s} \right) = \theta_R - \theta_L = \theta$$

This is the equation of Fry (1963, p.75)

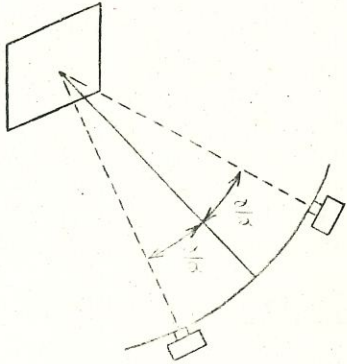
$$\frac{1}{e} = \frac{1}{\theta} \left( \frac{1}{r} - \frac{1}{s} \right)$$

$$e = \frac{\theta}{\frac{1}{r} - \frac{1}{s}} = \frac{\theta r s}{s - r} = \theta \frac{r}{d} (r + d)$$

The above is the equation of Colardeau (Judge, 1935, p.38) if  $\theta = 2\%$ , a parallax of  $1^\circ$ .

If  $r \gg d$ , then  $e = \theta \frac{r^2}{d}$ , which is the equation of Graham (1965, p.524) and the approximation of Fig. 17

Figure 21: Related parallax formulas



$\alpha$  = rotation angle  
 $w$  = height of scene  
 $\Delta$  = change in scene height due to keystoneing  
 $K$  = maximum size change between images which will still allow stereo vision

$\beta$  = angular height of scene  
 $\epsilon$  = angular width of scene  
 $D$  = distance to scene

For fusion,

$$\frac{2\Delta}{w} \leq K = 15\%$$

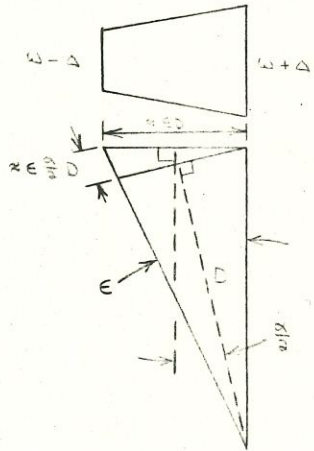
$$w = \beta D$$

$$(w + \Delta) - (w - \Delta) = 2\Delta = \epsilon \beta \frac{r}{\alpha} D$$

$$\therefore \frac{\epsilon \beta r}{\alpha} \leq K$$

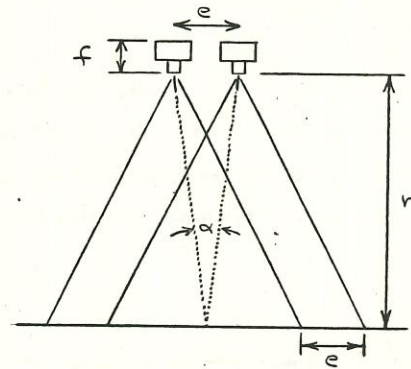
$$\epsilon \alpha \leq 2K$$

$$\alpha \leq \frac{2K}{\epsilon}$$

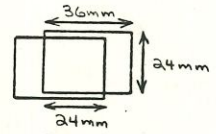


Example: 24x36 mm format, 50 mm focal length lens,  
 and a flat relief scene which fills frame  
 $\epsilon = \frac{36}{50}$ ,  $\max \frac{\alpha}{\epsilon} = \frac{2 \cdot 15}{\frac{36}{50}} = \frac{75}{36} \approx \frac{1}{5}$  radian  $\approx 11^\circ$

Figure 22: Maximum angle of rotation when photographing flat scenes before keystoneing becomes excessive



$\alpha$  = effective perspective difference angle  
 $e$  = camera translation or separation  
 $f$  = image focal distance, usually lens focal length



With 35mm film and  $\frac{2}{3}$  stereo overlap

$$\frac{e}{r} = \frac{12}{f}$$

Several values from this formula are listed:

f, mm	6	12	24	36	48	60	72	135
e/r	2	1	1/2	1/3	1/4	1/5	1/6	1/10
$\alpha$	90°	53°	28°	19°	14°	11°	10°	6°

Figure 23: Parallel camera alignment for flat scenes

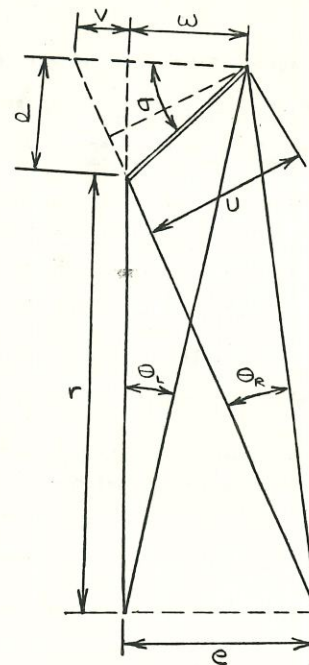
the scene slopes become flatter, a greater stereo separation can be used, even though the depth of the scene remains constant. Figure 24 illustrates the calculation of this effect; while the parallax decrease is small, it does give an extra safety margin to the 2° maximum parallax that has been chosen.

#### The Geometry of Perceived Space

Some writers on stereo photography have assumed that the distance to an object as perceived by viewing a three-dimensional scene or viewing stereo photographs is determined uniquely by the angle of convergence of the observer's eyes (Yacoumelos, 1972; Dewhurst, 1954, p. 31; Spottiswoode, 1953, p. 141). Experiment, however, has shown that subjective space is a distortion of the simple geometry of objective space, with the angle of convergence a poor depth cue (Ogle, 1962, p. 316; Graham, 1965, p. 520). An example of this is the fact that stereo photos can be viewed with a separation equal to the spacing of one's eyes. While the angle of binocular convergence will then be zero, the scene will not appear to be at an infinite distance (La Prade, 1972, p. 1178).

There have been many experiments for mapping our visual perception. For example, Gilinsky has shown that apparent distance is less than actual distance and that the difference increases with actual distance (Ogle, 1962, p. 256). Of greater interest for stereo photography is the variation of apparent depth with apparent distance (Fry, 1965, p. 75) and depth distortion as determined by the angles and distances of photography and viewing.

Stereo photography with camera separations greater than 65 mm is called hyperstereoscopy; it gives the viewer the effect of seeing a small nearby model of the real scene. In addition to the enhancement of depth perception which this effect allows, there can be an additional exaggeration of depth



$$\begin{aligned} \gamma &= \text{scene slope} \\ \theta &= \theta_R - \theta_L = \text{parallax} \\ \frac{de}{dr} &= \tan \gamma \\ \text{assume } \gamma &\approx 45^\circ, d \ll r \\ \theta_L &= \frac{3}{4} \text{ in radians} \\ \theta_R &= \frac{u}{\sqrt{r^2 + e^2}}, \frac{v}{u} = \frac{r}{\sqrt{r^2 + e^2}} \\ \theta_R &= \frac{(v+w)r}{r^2 + e^2} = \frac{\left(\frac{de}{dr} + w\right)r}{r^2 + e^2} \\ \theta &= \frac{\left(\frac{de}{dr} + w\right)r}{r^2 + e^2} - \frac{3}{4} \\ &= \frac{\left(\frac{de}{dr} + w\right)r^2 - w(r^2 + e^2)}{r(r^2 + e^2)} \\ &= \frac{der - we^2}{r(r^2 + e^2)} \end{aligned}$$

Substituting for  $w$ ,

$$\begin{aligned} \theta &= \frac{de}{r(r^2 + e^2)} \left( r - \frac{e}{\tan \gamma} \right), \theta \uparrow \text{ as } \gamma \uparrow \\ \text{for } \gamma &= 90^\circ, \theta = \frac{de}{r^2 + e^2} \\ \text{for } \gamma &= 45^\circ, \theta = \frac{de}{r^2 + e^2} \left( 1 - \frac{e}{r} \right) \end{aligned}$$

Figure 24: Scene slope and angular parallax

which alters the apparent slopes of the scene. This second effect is called a shape ratio (Spottiswoode, 1953, p. 37); it is the ratio of depth magnification to width magnification. It causes cubes to appear flattened or extended in the direction of viewing.

When a stereo pair of photographs is viewed with an angle of binocular convergence of  $11^{\circ}$ , this magnification of slope is approximately five times the ratio of stereo separation to the distance to the scene (La Prade, 1972; Yacoumelos, 1973). This indicates that slopes will be somewhat flattened when viewing photos of a scene photographed with an angle of convergence of the stereo camera less than  $11^{\circ}$ .

Depth exaggeration occurs also when one views an actual scene. Here, the separation of one's eyes remains constant; it is found that the apparent slopes of scenes decrease with distance (La Prade, 1973, p. 38). Therefore, depth exaggeration when viewing stereo photos is possibly similar to that of viewing real scenes.

A surface around one's direction of fixation whose points appear by stereopsis to be the same distance from the observer is called a horopter (Ogle, 1962, p. 352). This surface is not usually a plane perpendicular to the observer's line of sight. Instead, Kroncke has found that this surface is usually concave toward the observer for viewing distances less than about one meter and concave away from the observer for greater distances (Le Grand, 1967, p. 237).

A similar nonlinearity exists in a direction along one's line of sight. In what is called the alley experiment, Blumenfeld asked his subjects to arrange two lines of lights so that they formed apparently parallel lines extending directly away from them (Graham, 1965, p. 538). The actual arrangement of the lights was found to be a pair of diverging curved lines.

These relationships between objective and subjective visual space have been summarized and combined with mathematical transforms (Luneburg, 1947). However, these equations appear to imply a greater accuracy than the meager experimental data contain (Ogle, 1962, p. 395).

The ancient Greek architects evidently were familiar with the correction of subjective visual distortions. While it is possible that the idea of entasis was a remnant of much earlier construction (Marquand, 1909, p. 89), there are a number of other independent curvilinear deviations from straight and perpendicular lines found in Greek architecture. Examples of this are the curvature of the stylobate and entablature and the upward convergence of the axes of columns in Greek temples (Anderson, 1927, p. 119). While it is possible that these variations were simply to make the forms pleasing visually, it is also possible that a definite attempt was made to create straight and perpendicular lines in the viewer's subjective perception.

When viewing stereo photos, it sometimes happens that their depth information is overridden by one's prior experience. For example, a hollow mask of a human face can sometimes look like a normal face (Gregory, 1970, p. 128). A related effect is due to the fact that one is used to seeing shadows cast below an object or toward the observer; when stereo photos are viewed with the shadows in the photos falling away from the viewer, relief can be reversed so that depressions become elevations (Howard, 1970, p. 149; American Society of Photogrammetry, 1952, p. 527). In some fairly artificial stereo views, stereopsis can give only ambiguous depth information (Julesz, 1969, p. 591; Julesz, 1971, p. 186).

#### Testing Stereoscopic Vision

Roughly 10-15% of the population has weak stereo vision. While it is

difficult to detect deficient stereo vision when normal photographic scenes are viewed with a stereoscope, one published test example allows a good qualitative evaluation of stereopsis (Avery, 1968, p. 33). Other, quantitative, stereopsis tests allow comparisons between individuals (Howard, 1970, p. 150; Hallert, 1960, p. 59; Julesz, 1971, p. 270). Stereoscopic acuity can be improved by training; it also usually improves during a stereo viewing session (Howard, 1970, p. 143).

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DR. RAINEY

STEREO PHOTOGRAPHY  
FOR THE ARCHAEOLOGIST

An Extra Dimension for Records and Education

A Report from the Museum Applied Science Center for Archaeology

The University Museum  
University of Pennsylvania  
Philadelphia, Pennsylvania

Bruce Bevan

15 May 1973

AN INTRODUCTION TO STEREO PHOTOGRAPHY

Bruce Bevan

Stereoscopic photography will add the dimension of depth to your illustrations. This report describes the basic technique of photographing landscapes, buildings and smaller objects in three dimensions.

For stationary objects, a single, standard camera is all that you need to reconstruct a three-dimensional view with stereo photography. Take two pictures of the scene you want to record, each from a different viewpoint:

- 1: Center the scene in the camera's viewfinder
- 2: Photograph the scene
- 3: Move left or right a short distance
- 4: Again, center the scene in the viewfinder
- 5: Photograph the scene again

The selection of the distance between the two points at which the pair of stereo photos are taken is described next. When you photograph distant scenes, the separation of these photo points will be greater than the distance between your pair of eyes; then, when you view the pair of photos, your perception of the depth of the scene will be enhanced.

In the scene which you will photograph, look for a point where the foreground and background of the scene appear to be near each other from your point of view; a tree branch against the horizon or a wall almost perpendicular to the picture are two examples. As you move sideways, this foreground point will shift sideways relative to the background.

Hold a pencil in your hand and stretch your arm straight out as shown in Figure 1. Select the photo points for your stereo pair such that the shift of the foreground relative to the background that you see with one eye closed is about equal to the "width" of the pencil.

Many scenes have no abrupt step between their near and far points. Fortunately, the accuracy of stereo separation is even less important for these scenes, and you can usually estimate closely enough. Also, you can use the length of the pencil to compare the shift between vertically separated foreground and background points in these more rounded scenes.

This technique is accurate and easy to use, but is not applicable if your camera has a wide angle or telephoto lens. The following methods will also let you approximate stereo separation; with them, you may use any type of camera or lens that you have.

For objects such as buildings and statues, the separation of the photo points should be about 5% of the width of the object; see Figure 2. However if only part of a large object is being photographed, measure only the width that will be in the picture. Except for close-up photography, when the object is less than 25cm from the camera, make the stereo separation 6cm or greater. An accuracy of 50% in the determination of the separation of photo points is all that is required.

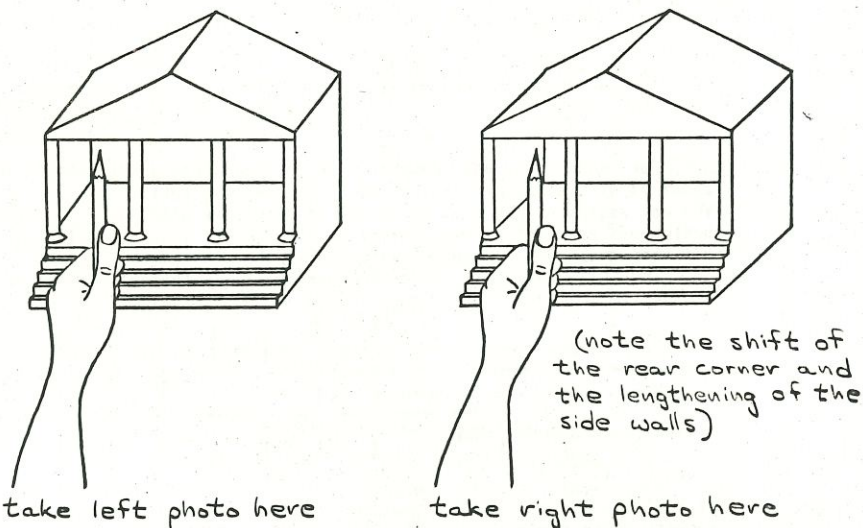


Figure 1: The pencil as a stereo measure

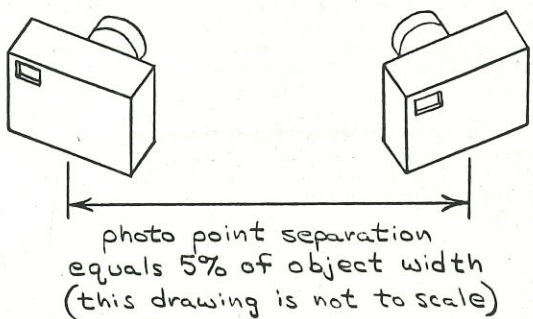
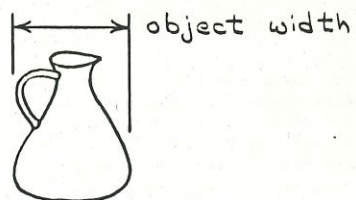


Figure 2: Stereo photography of objects

STEREO PHOTOGRAPHY FOR THE ARCHAEOLOGIST

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For landscapes, the distance between your photo points should be about 2% of the width of the scene at the nearest part of the photo. You can estimate this from the width of the scene that you see at the bottom of the camera's viewfinder; an accuracy to within 50% is again all that is needed. Figure 3 illustrates the procedure.

The following table can be used as an aid in the calculation of the stereo separation:

Width of the scene		100	200	300	400	500	600	700	800	900
Distance between stereo photo points	Landscape	2	4	6	8	10	12	14	16	18
	Object	5	10	15	20	25	30	35	40	45

For example, a landscape scene 400 feet wide as seen at the bottom of the camera's viewfinder should have a photo point spacing of about 8 feet; also, for a statue 65cm wide, a 3cm stereo separation is indicated, but this should be increased to the minimum 6cm.

After film processing, you can view your pair of stereo photo prints by placing them under a lens viewer as illustrated in Figure 4. Arrange the prints in the same left-right order as they were photographed. The distance between matching points on the pair of photos should be about 55mm; therefore, the width of the photos can also be no greater than 55mm.

By making minor adjustments in the positions of the photos and the stereoscope, you can align the two photos relative to your pair of eyes and thereby eliminate image doubling when you look through the twin magnifying lenses. You can verify this by closing one eye and then the other; the image should not jump. Try to view a sharp, contrasty part of the scene first. While it may take a minute before your eyes adjust to seeing stereo this way, the startling effect of the depth that you can see will be worth the effort.

Stereo photos which are color or black and white transparencies can also be viewed. Hold two small slide viewers together to create a makeshift lens stereoscope; alternatively, the strips of film can be mounted for viewing in commercially-available stereo slide viewers.

For a more detailed discussion on stereo photography, see the companion report to this one, "Stereo Photography for the Archaeologist", dated 15 May 1973.

Museum Applied Science Center for Archaeology  
The University Museum  
University of Pennsylvania  
Philadelphia, Pennsylvania  
26 October 1973

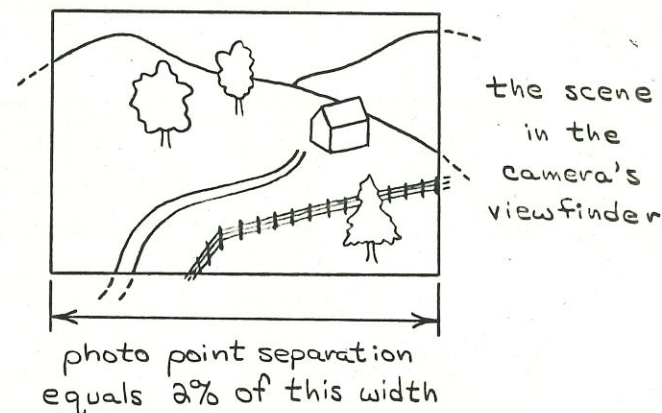


Figure 3: Stereo photography of landscapes

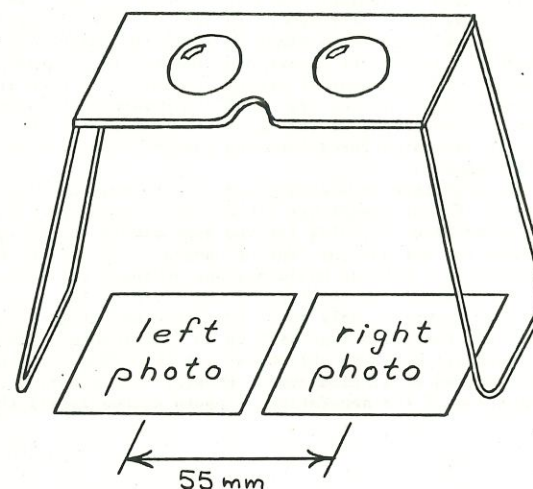


Figure 4: A lens viewer for stereo photos

## STEREO PHOTOGRAPHY FOR THE ARCHAEOLOGIST

### PART I: THE PRACTICE OF STEREOSCOPIC PHOTOGRAPHY

#### The Purpose of this Report

Three-dimensional illustrations are an aid to teaching and communication. Stereo photography adds the dimension of depth. It will improve the viewer's comprehension of most scenes; furthermore, stereophotography is necessary to show the true three-dimensional structure of some scenes. This report outlines the techniques of stereo photography of excavations, statues, small finds, and architecture. The emphasis is entirely with the application of stereo photography for illustration and recording and not for mapping or plan making from photographs.

This report outlines the procedure of taking stereo photographs by sequential exposure with one camera: taking a picture of a scene, moving left or right a short distance, and taking a second picture of the same scene. The separation of these photo positions will sometimes be a greater distance than the separation of one's eyes. A stereoscopic view of the resulting pair of photos will give the effect of having an effective eye separation greater than that which is humanly possible; this will enhance depth perception.

#### The General Principles of Stereo Photography

Stereoscopic vision and photography are based on the principle that the appearance of three-dimensional scenes is different from separated viewpoints; that is, the foreground of a scene is displaced relative to its background when viewed from two points. This phenomenon is called parallax. Figure 1 illustrates the parallax one sees when looking down on the apex of a pyramid.

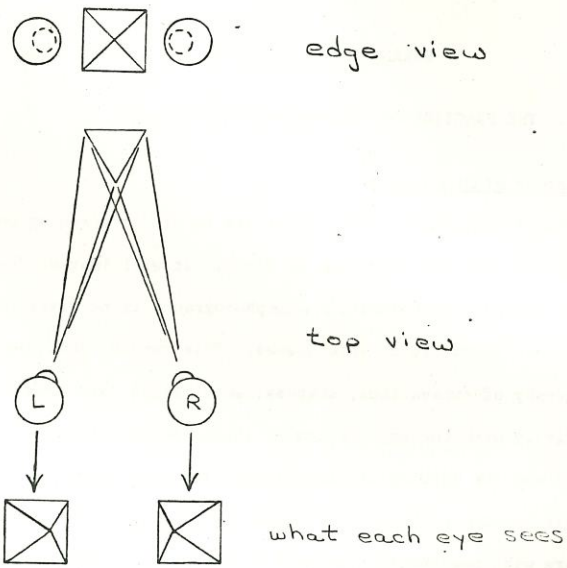


Figure 1: Binocular parallax of a pyramid

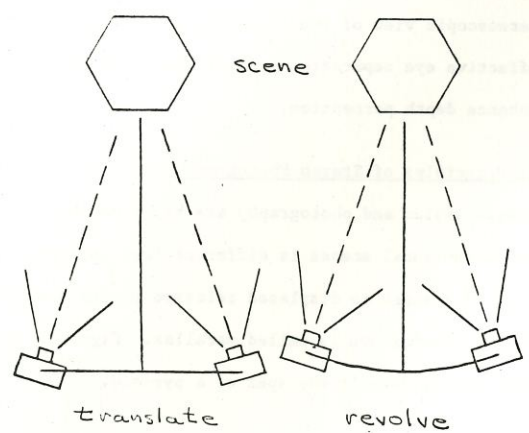


Figure 2: Converging alignments of camera positions

The two viewpoints which humans normally have are determined by an eye separation of about 65mm. With stereo photography, other viewpoint separations are possible. With a small separation the difference in the scene from the two viewpoints can be too small for three-dimensional depth perception. However, if the stereo separation is too large, the eyes and brain cannot combine the two dissimilar images and the scene is just an eye-straining confusion.

Many photographers who take three-dimensional pictures always use a photo point separation approximately equal to one's eye separation. Either they use commercially available stereo cameras or they use a general purpose camera and take a stereo pair by photographing a scene, shifting their weight from one foot to the other to move the camera slightly, and then photographing the scene again. A more valuable and vivid 3-D view requires an understanding of maximum photo point separation as it is determined by the dimensions of the scene and the parallax limits of easy stereo viewing.

Photo point separation is determined primarily by the depth of the scene and the distance to the scene. The depth of the scene is determined by the region of interest for 3-D viewing. It can be the depth of a statue or building as it is visible from the photo point; in landscape photography, it can be effectively infinite if the far horizon or clouds are important to the picture. Sometimes the foreground and background of the scene of interest do not need to be included in the determination of the depth of scene if they are out of focus or have a plain or unobtrusive texture, such as a bare floor or wall or a clear sky. Even if this is not the case, it is sometimes necessary to insure

adequate depth perception in the scene of interest by deliberately causing excessive parallax in the foreground or background. The distance to the scene is simply the distance from the photo point to the beginning of the scene of interest.

The procedure for stereo photography is as follows: frame the scene of interest in the camera and take the first photo, move left or right for scene parallax keeping the distance to the scene approximately constant, frame the scene of interest approximately as it was in the first photo by checking tilt and centering, take the second photo.

Photo point separation allows one to control scene parallax and therefore depth perception. In later sections of this report the method of determining photo separation for specific types of scenes will be explained. There is one general rule from which all these specific applications are derived: make sure that scene parallax is less than  $2^{\circ}$ . That is, when one moves sideways for the stereo pair make sure that the near point in the scene moves not more than  $2^{\circ}$  relative to the far point in the scene. For average scenes one will probably want to move so that the parallax is one half or one quarter of this value. If this maximum is exceeded it will become more difficult to see the depth of the scene when the photos are viewed because of their excessive stereo dissimilarity.

Scene parallax is easiest to determine when there is a sharp edge between the near and far points in the scene, which is often the case with architectural photography. A convenient angular measure is the width of the thumb on one's outstretched hand; this can be approximately  $2-3^{\circ}$ . To apply this "rule of thumb" when photographing a building, use a stereo separation such that a front corner of the building moves

relative to the background approximately half the width of the thumb as one moves sideways between photo positions. This rule is more difficult to apply to scenes that are rounded or that staircase with depth; fortunately, stereo separation is also less critical with these types of scenes.

One can easily measure the angular width of one's thumb by taping a meter stick to a wall at eye level and standing so that the tips of one's toes are 57 cm from the wall. The thumb on one's outstretched arm will cover a length of the meter stick; the apparent width of the thumb in centimeters is then equivalent to its angular width in degrees.

There is one general precaution one must take with this type of sequential stereo photography, and that is, there must be no movement within the scene during the time between taking the pair of photos. Watch out for moving people or wind-blown trees; also, if one waits more than several minutes between photos, there can be noticeable movement of shadows. One must also be sure that the stereo pairs are taken from approximately the same elevation. Never move obliquely up or down between photos unless the alignment of the camera frame is oblique also. While it is possible to correct for misalignment when the photos are printed or viewed, one will save film by starting correctly.

If moving objects are to be photographed, the shutters on two cameras must be synchronized to release at the same time. If the separation of the two cameras is greater than about two meters, it will probably be necessary to use separate tripods with electrical solenoids used to trigger both cameras simultaneously. With a separation of one meter or less, the two cameras can be mounted on a rigid bar, and both can be triggered with a paired cable release. Since precise synchronization of the two cameras will be difficult, objects which are moving very fast cannot be photographed.

#### How to Photograph Statues and Architecture

This type of stereo photography is about the easiest to do and the most fun to view. The subjects are usually medium relief scenes for which the distance to the scene is often in the range 2-20 times the depth of the scene. One can usually frame each of the pair of stereo photos the same and use a converging alignment of camera aim

directions as Figure 2 shows. However, for some architectural facades one must follow the technique outlined in the section on flat, low relief scenes in order to minimize horizontal perspective shift or keystoneing.

As an example of this type of photography, consider the case of a statue in front of a blank wall as shown in Figure 3. The depth of scene here is just the depth of the statue visible in the direction of view (if the wall had a strong pattern, the depth of scene would be the distance from it to the front of the statue). If the distance from the camera to the statue is five times the apparent depth of the statue, then the maximum camera separation is equal to the apparent depth of the statue. In most cases it would be best to use about half of this maximum stereo separation.

For other photo situations one can follow the rule that the ratio of stereo separation to scene depth varies roughly with the square of the ratio of the scene distance to depth as tabulated in Figure 3. Note that the unit of measurement can be anything convenient: feet, meters, or paces.

If distances are to be estimated from the position of the camera, remember that it is possible to estimate tangential distances more accurately than radial distances from one's viewpoint. Since these two measurements are related by an angle, it is possible to use the "rule of thumb" once more. In my case, if the thumb on my outstretched arm covers a strip of the scene which appears to be one meter wide, my distance to the scene is about 25 meters. The distance that one's thumb appears to shift across the scene when one looks at the scene with first one eye and then the other can be used also as a parallax range-

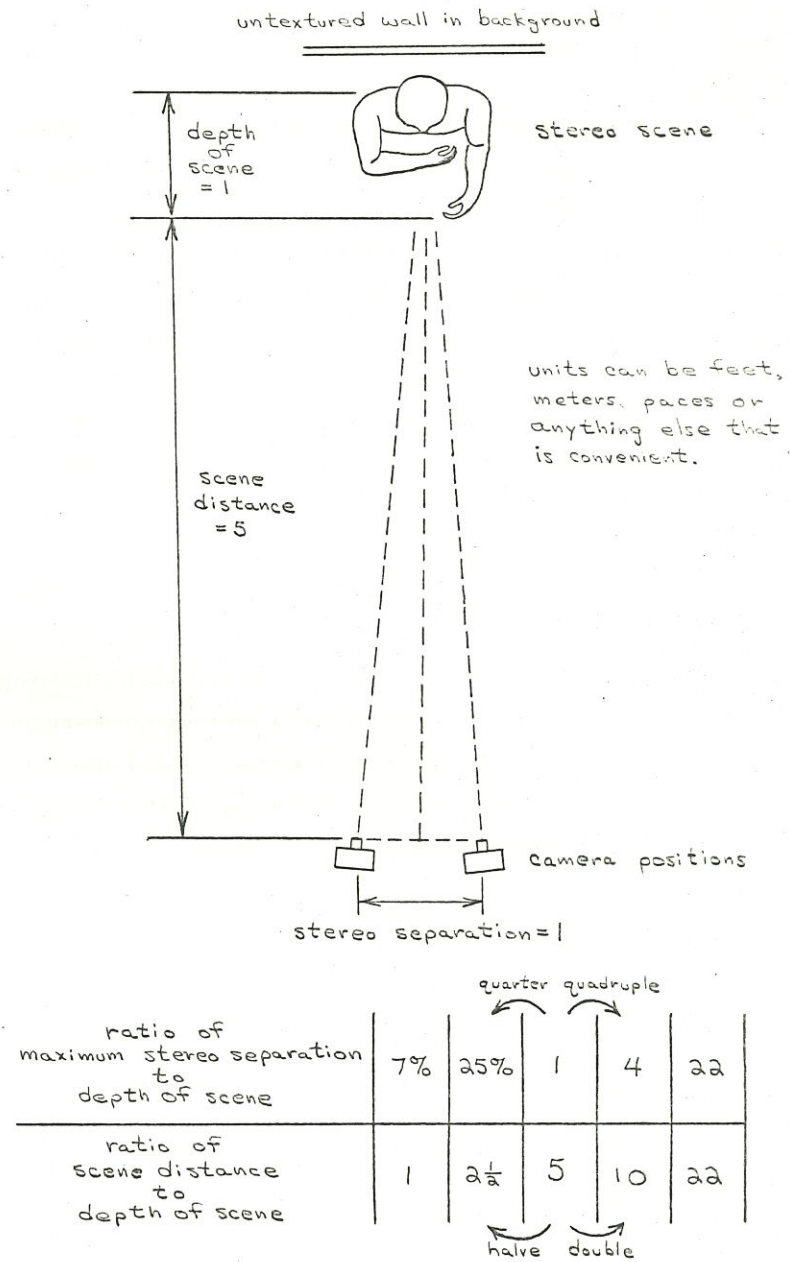


Figure 3: An example of stereo separation,

finder. My proportionality for this measure is nine, so that in the above example my thumb would appear to jump a little less than three meters. If one stands a measured distance from a meter stick on a wall, one can measure his own rangefinder constants.

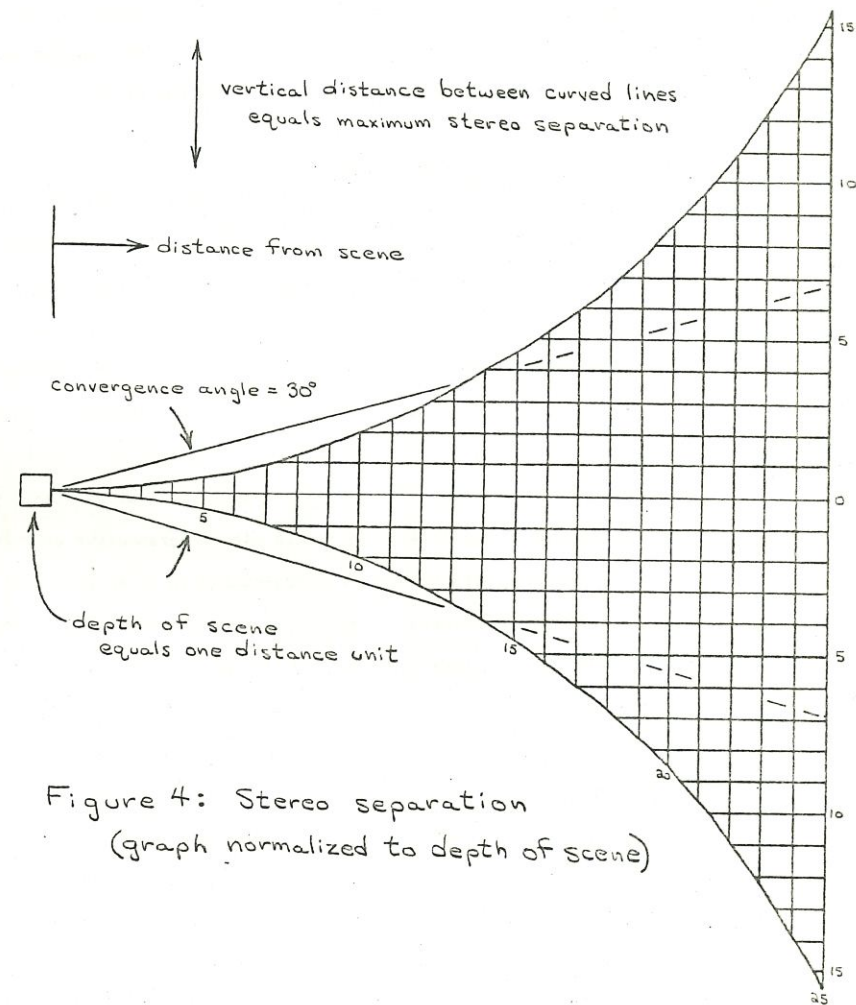
A graph for determining maximum stereo separation is given in Figure 4. Both distance to the scene and stereo separation are normalized to the depth of the scene in this graph. For example, if the distance to the scene is 8 times the depth of the scene, this graph indicates a maximum stereo separation of about  $2\frac{1}{2}$  times the depth of scene. Stereo photography with the pointing directions of the camera converging at angles greater than  $30^\circ$  should be avoided in general, for the convergence of the eyes required for viewing can be difficult. (Dalzell, 1936, p. 198).

The purpose of these procedures for determining stereo separation is simply to get one acquainted with proper stereo separation in several applications; with just a little experience one is able to estimate stereo separation well enough for most applications by simply observing the parallax of the scene.

#### How to Photograph Excavations

Landscape scenes are usually recorded with oblique photographs and therefore frequently have a depth of scene greater than the distance to the scene. If nearly vertical photos are taken from an elevated platform, follow the rules given in the preceding or following sections.

With oblique stereo photography, one's maximum stereo separation should be about 4% of the distance to the nearest point of the scene of



interest, as Figure 5 illustrates. In many cases a small camera separation may be used. If the bottom edge of the camera frame intersects the scene at  $1\frac{1}{2}$  meters from the camera, the maximum stereo separation is reduced to an interocular separation of 65 mm.

#### How to Photograph Flat, Low Relief Scenes

In this category bas-reliefs, inscriptions and coins are included. A large stereo separation is required to make the relief of these flat scenes visible. In the previous examples it has been possible to keep the camera centered on the scene for the pair of stereo photos. In this case excessive keystoneing, the distortion of squares into trapezoids, can result. In Figure 6 the difficulty with a rectangular scene is illustrated (Valyus, 1962, p. 174).

While a slight amount of keystoneing will still allow stereo viewing, keystoneing can be completely eliminated with a perspective correction camera. This requires either an expensive camera or an expensive special lens for a standard camera. Fortunately, one can do almost as good a job by keeping the camera aligned parallel to the flat scene instead of inclining it. Figure 7 illustrates this method.

The easiest way to do this with a 35 mm camera which has a lens with normal focal length is as follows: position the camera so that the scene fills the right hand two-thirds of the frame and the camera is looking perpendicular to the scene. Take a picture and then move right a maximum of 25% of the distance to the scene. Then place the scene in the left hand two-thirds of the frame. Again, make sure the

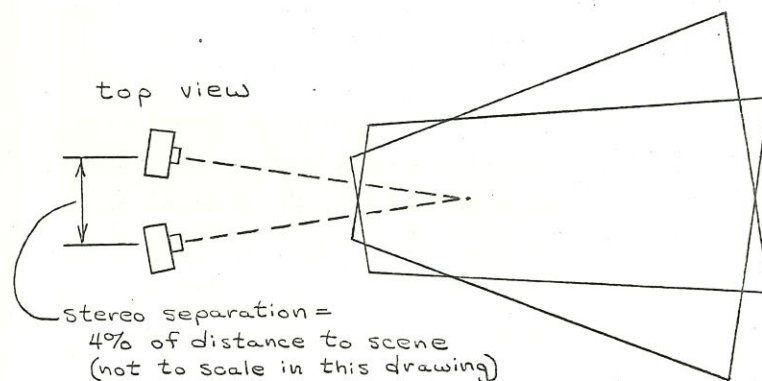
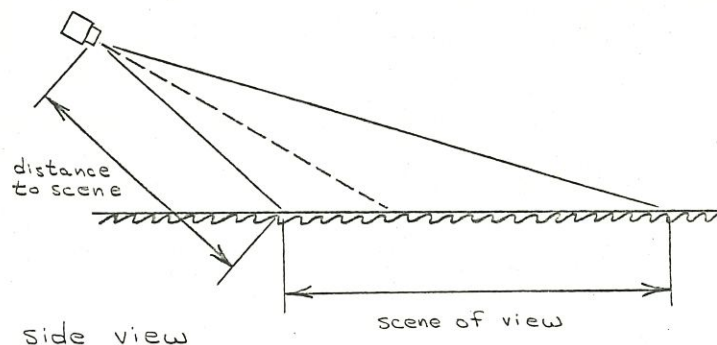
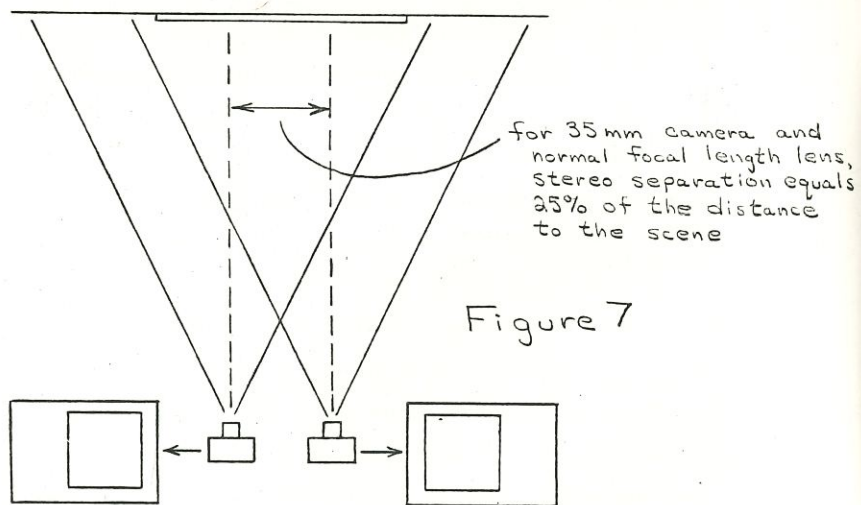
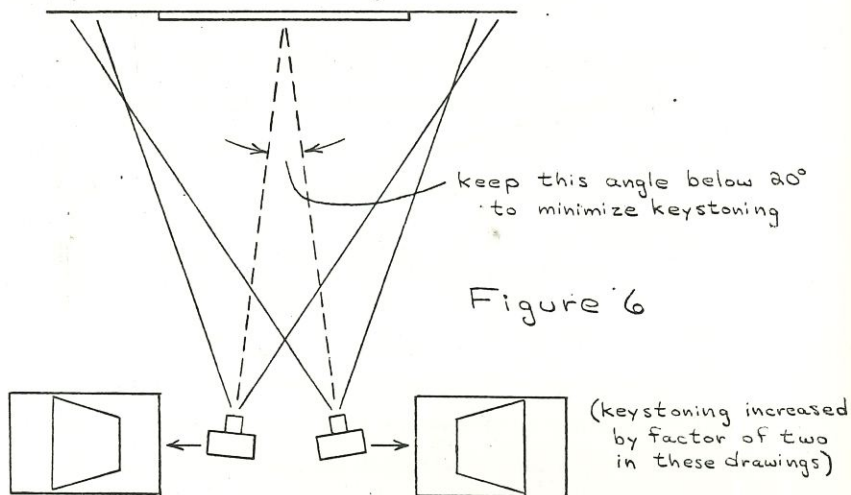


Figure 5: Oblique stereo photography



Photography of flat, low relief scenes

camera is perpendicular to the scene and take the second picture. A portion of each frame will furnish a stereo view of the scene common to both photos.

The stereo separation for a square stereo overlap varies with the focal length of the lens as given in this table for 35 mm cameras.

Focal Length of Lens	Stereo Separation as a Percentage of the Distance to the Scene
24	50%
48	25%
60	20%
120	10%

There is one general rule which is independent of the lens. When looking at the scene through the camera's viewfinder, estimate the width in the scene of one third of the frame. This will be the stereo separation. It is possible to use less stereo overlap if one wishes greater depth enhancement, but one will also be wasting more of the available area of the film.

The stereo photography of coins or intaglios is easy with a single lens reflex camera. Select a lens for close-up work and mount the camera on a stand. Frame the object at one end of the viewfinder and take the first photo. Slide the object, without rotating it, so that it is in the other end of the viewfinder and take the second picture. The separation of these photo viewpoints will often be less than the spacing between one's eyes.

#### How to Photograph Small Objects

While small, low relief objects have been discussed in the previous section, other special techniques can be applied to items such as stone tools

and terra-cottafinds which can also be moved easily. The information on photographing statues given before can sometimes be applied to these objects.

Here again, the depth of scene is important. Note that for a cylindrical object with a flat-textured background the depth of scene is usually the radius of the cylinder, not the diameter.

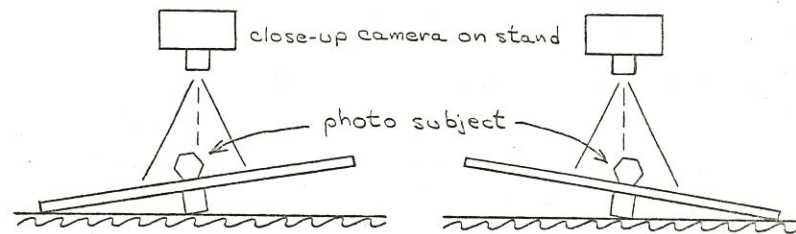
Since with small objects it is often easier to rotate the object than move the camera, the following table will help to estimate the best angle. Be careful with angles of rotation greater than about  $30^\circ$ ; since this can exceed the normal angle of convergence of one's eyes, there can be difficulty in viewing the photos.

Ratio of Scene Distance to Scene Depth	1	2	5	10	20
Maximum Angle of Rotation	$2^\circ$	$4^\circ$	$10^\circ$	$20^\circ$	$40^\circ$

If one rotates the object in front of the camera, a plain background must be used. In most cases the object must be illuminated by diffuse light (Kodak, 1969, p. 61) or the lights must rotate with the object in order to prevent confusing shadows from causing difficulty with stereo viewing (Symons, 1969, p. 1468).

One can take a sequence of photos around an object by putting it on a turntable which has a protractor for setting angles; later one can view adjacent pairs of photos.

A very easy way to rotate small objects is with a tilting table (Wienert, 1960). Figure 8 illustrates how this is done. A ridge under a flat board allows the object to be rotated easily through an angle determined by the design of the table. Since the angle of rotation is not critical, a single tilt-table



ratio of ridge height to total table width	0.87%	1.7%	3.5%	4.4%	6.6%	8.1%	13.4%	18.2%
rotation angle	$2^\circ$	$4^\circ$	$8^\circ$	$10^\circ$	$15^\circ$	$20^\circ$	$30^\circ$	$40^\circ$

For example, if a terra cotta fragment with 2 cm of relief is photographed from a distance of 20 cm, a rotation angle of about  $15^\circ$  would be suitable. A 20 cm square tilt table with a 1.3 cm square cross section rod under it could be used.

Figure 8: A tilt-table for small finds

can be used with specimens of somewhat different sizes. One can design a more elaborate table to vary the angle of rotation or to minimize the horizontal displacement of the object when it is tilted.

#### Viewing Stereo Photographs

Some people unfortunately do not have good enough stereo vision to enjoy viewing stereo photographs. About 2% of the population has no stereo vision at all and another 15% has only weak stereo vision (Julesz, 1971, p. 270; Howard, 1970, p. 143).

Many people, however, have very acute stereo vision and can perceive the separation in depth of an object at  $\frac{1}{2}$  kilometer against a far horizon (Rabben, 1960, p. 140; Hallert, 1960, p. 57). Stereoscopic acuity for both far and near objects can be enhanced by "spreading one's eyes" more than the average 65 mm spacing we have (Nowicki, 1952, p. 524; Howard, 1970, p. 143).

With photography we can easily select any amount of depth enhancement we wish. For example, if we take photos with a camera separation of ten times the separation of our eyes and then view the films, we will seem to be looking at a model of the real scene that is roughly ten times nearer to us and ten times smaller (Purves, 1969, p. 762; Hallert, 1960, p. 64) and our depth acuity is increased also by this factor. In aerial photography, the distance between photo points can exceed 2 km giving a depth enhancement of over 30,000 times (Rabben, 1969, p. 141; Hallert, 1960, p. 65).

Most people cannot look at stereo photos without optical aids. This is because of the difficulty we have in trying to keep each eye on separate photos; our eyes tend to converge to one photo (Gregory, 1970, p. 125; Avery, 1968, p. 25).

The simplest way to view stereo photos is with a lens stereoscope. This instrument consists of a pair of 2x to 3x magnifying lenses separated by the

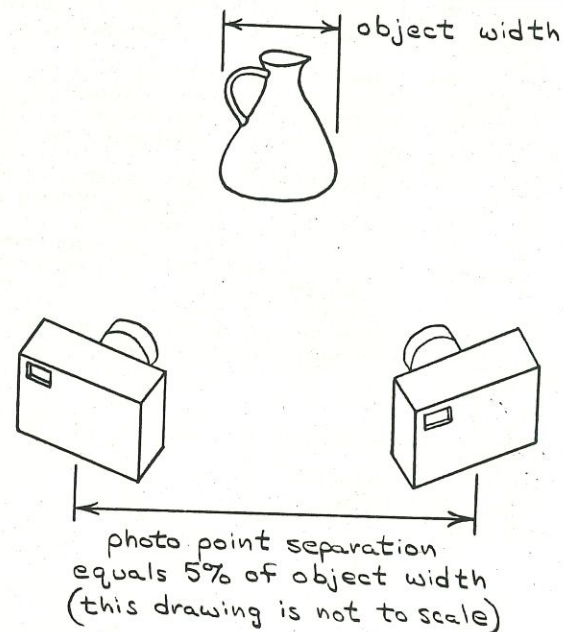
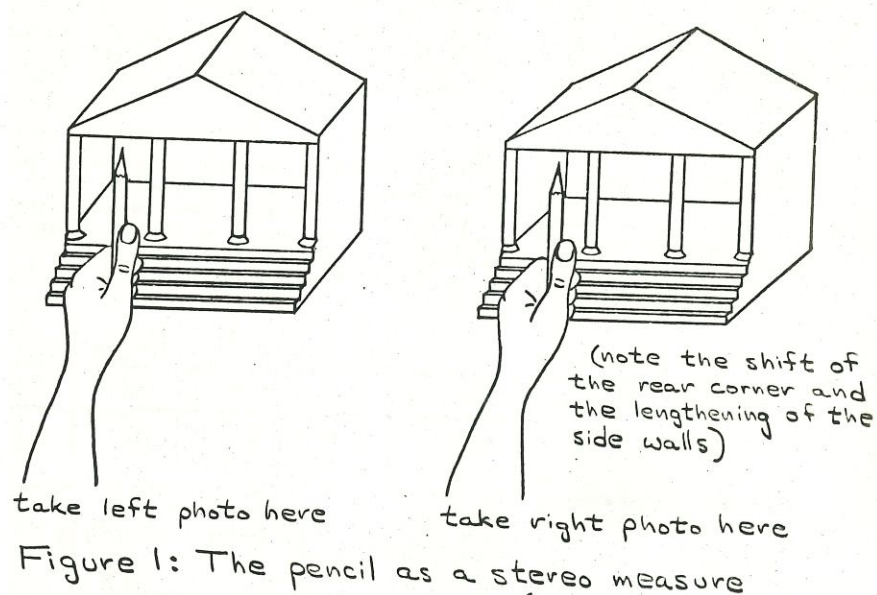


Figure 2: Stereo photography of objects

## AN INTRODUCTION TO STEREO PHOTOGRAPHY

Bruce Bevan

Stereoscopic photography will add the dimension of depth to your illustrations. This report describes the basic technique of photographing landscapes, buildings and smaller objects in three dimensions.

For stationary objects, a single, standard camera is all that you need to reconstruct a three-dimensional view with stereo photography. Take two pictures of the scene you want to record, each from a different viewpoint:

- 1: Center the scene in the camera's viewfinder
- 2: Photograph the scene
- 3: Move left or right a short distance
- 4: Again, center the scene in the viewfinder
- 5: Photograph the scene again

The selection of the distance between the two points at which the pair of stereo photos are taken is described next. When you photograph distant scenes, the separation of these photo points will be greater than the distance between your pair of eyes; then, when you view the pair of photos, your perception of the depth of the scene will be enhanced.

In the scene which you will photograph, look for a point where the foreground and background of the scene appear to be near each other from your point of view; a tree branch against the horizon or a wall almost perpendicular to the picture are two examples. As you move sideways, this foreground point will shift sideways relative to the background.

Hold a pencil in your hand and stretch your arm straight out as shown in Figure 1. Select the photo points for your stereo pair such that the shift of the foreground relative to the background that you see with one eye closed is about equal to the "width" of the pencil.

Many scenes have no abrupt step between their near and far points. Fortunately, the accuracy of stereo separation is even less important for these scenes, and you can usually estimate closely enough. Also, you can use the length of the pencil to compare the shift between vertically separated foreground and background points in these more rounded scenes.

This technique is accurate and easy to use, but is not applicable if your camera has a wide angle or telephoto lens. The following methods will also let you approximate stereo separation; with them, you may use any type of camera or lens that you have.

For objects such as buildings and statues, the separation of the photo points should be about 5% of the width of the object; see Figure 2. However if only part of a large object is being photographed, measure only the width that will be in the picture. Except for close-up photography, when the object is less than 25cm from the camera, make the stereo separation 6cm or greater. An accuracy of 50% in the determination of the separation of photo points is all that is required.

distance between one's eyes. Make contact prints of black and white 35 mm stereo photos and place a pair under the lens stereoscope. Separate the photos so that the distance between corresponding image points in the pair is about 55 mm (Avery, 1968, p. 26; Nowicki, 1952, p. 531; Howard, 1970, p. 149); note that this is less than the separation of one's eyes. Be sure that the right hand photo that was taken is to be viewed by the right eye; otherwise one will see an impossibly inside-out confusion. With a little juggling of the viewer and photos one should be able to get the two photos to fuse into one 3-D scene.

Since the maximum usable width of a photo viewed with a lens stereoscope is 55 mm, 35 mm or 2 $\frac{1}{4}$  inch format cameras are excellent for stereo photography. Also, for example, note that a camera with a 4x5 inch or larger format would require that its photos be cropped or reduced in size during printing.

One can also make 2x to 3x enlargements of the negatives but one may have to crop them slightly to furnish the 55 mm separation, or one can view the enlargements by bending one of the photo edges temporarily upward so that the photos do not overlap when viewing near the center of the photos. Note that it can be impractical to dodge or burn-in while printing enlargements because of the difficulty of treating both halves of the stereo pair the same.

Correct for stereo misalignment by cropping enlargements and framing the stereo image relative to the edges of the photo (Kingslake, 1951, p. 192; McKay, 1948, p. 55; Kaiser, 1955, p. 76; Dalzell, 1936, p. 163). Stereo viewing gives one the impression of looking through a window at a three-dimensional scene. Except for some dramatic purposes, it is best to insure that the scene is entirely on the far side of the window resulting from the photo's framing. In order to do this, crop the left hand photo of the pair

to include the scene of interest; make it less than 55 mm wide but any desired height. Locate the nearest point to the observer in the photo and then crop the right hand photo so that this point is the same distance from the left and right edges in both photographs. This will put the nearest point in the photo right on the "window pane." To move the entire scene slightly behind the window crop the right hand photo a slight amount to the left of the position indicated above. This idea is illustrated in Figure 9.

One can make a stereo viewer for color slides by simply fastening together two inexpensive slide viewers. These usually have an adjustable focus on their lenses and diffusing screen behind the slide. Since it will probably be impossible to bring the two viewers close enough together so that the slide images are separated by 55 mm, one will have to duplicate a 100-year-old invention known as the Brewster Stereoscope (Purves, 1969, p. 1457). As Figure 10 illustrates, vary the spacing and angle of the viewers to take advantage of the prismatic effect of the lens to enable one to fuse visually the more widely separated photos.

Since one cannot make adjustments easily in the positions of the slides while they are in this viewer, it is important to frame the scenes accurately when they are photographed. However, for casual viewing, one's eyes will readily adjust to minor misalignments.

Most stereo viewers have lenses with focal lengths that are approximately equal to the focal length of a normal lens on a 35 mm camera, about 50 mm. When viewing contact prints or transparencies each of one's eyes will then see normal perspective (Kodak, 1967; Larmore, 1965, p. 179; Purves, 1969); this has been the basis of the rules given before on stereo separation. If a telephoto lens is used for stereo photography, determine the stereo separation by assuming that the actual distance to the scene is divided by the ratio

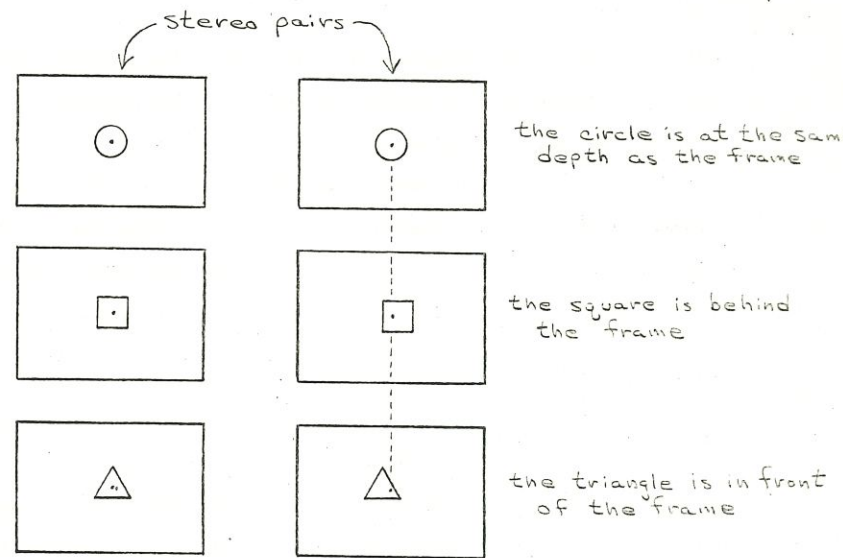


Figure 9: Framing stereo photographs

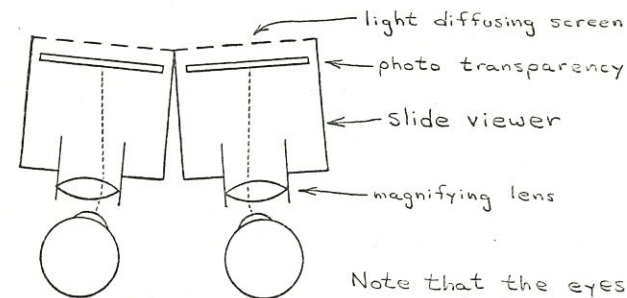


Figure 10:  
A stereo slide viewer

Note that the eyes look through the inner edge of each lens. Most slide viewers require this prismatic effect for convergence of the eyes.

of focal length of the telephoto lens to the focal length of a normal lens. In a related manner, it is possible to increase stereo separation if one uses a wide-angle lens.

If one plans to enlarge the photos before viewing them, remember that this is equivalent to using a telephoto lens. However, enlargements can often be viewed with correct perspective by using a mirror stereoscope (Nowicki, 1952, p. 528).

#### Publishing Stereo Photographs

A stereo photograph provides an excellent way of illustrating a site and artifacts more clearly in publications. For example, with some dark colored or shiny objects such as obsidian tools, it is very difficult to show three-dimensional shape in one photo. With color photography, objects must often be photographed in flat, diffuse lighting that eliminates the shadows which give good depth cues in a single photograph. Both of these cases can be helped by publishing stereo photographs.

While some archaeologists will have a lens stereoscope handy for viewing published photos, one must consider including viewing equipment with the report. Flat, Fresnel lens stereoscopes are made of inexpensive and flexible plastic and can be included with an article (Julesz, 1960). Cardboard and plastic viewers for classroom use or publication are available from Taylor-Merchant Corporation, 24 West 45th Street, New York City 10036. Because of the magnification of the lenses of these viewers, the half-tone dots on the printed photo may be visible; this is objectionable, but not serious. It is possible to publish color stereo photos with this method; however, the photos must be less than about 50 mm wide.

Most people find it easiest to view stereo photos as anaglyph prints. In this process the stereophotos are printed, one red and one green, directly on top of each other (Dudley, 1965, p. 96; Nowicki, 1952, p. 526; Hallert, 1960, p. 62; Valyus, 1962, p. 188) and viewed with a red celluloid over one eye and a green celluloid over the other. While the viewer will be quite inexpensive, a quality, two color print will add to the expense. When good color control is used for both the printing inks and the celluloid filters, the viewing is excellent (Julesz, 1968; Gregory, 1970; Fejes Toth, 1964; Kosofsky, 1970). Anaglyph viewers are available in large quantities from Brownie Manufacturing Company, 261 Broadway, New York City 10007. One may publish a photo of any size with this method, but it must be black and white originally.

A third method has, I believe, been too little used; this is the single mirror type of stereo viewer (Julesz, 1967; Purves, 1969, p. 1457). With this technique, one photo of the stereo pair is reversed left-to-right in printing and viewed in a mirror's reflection as shown in Figure 11. A normal glass mirror cannot be used, but an inexpensive plastic mirror which reflects only from its front surface could easily be included with the report. This technique can be used with a photo of any size and also with color prints.

#### Other Related Topics

Underwater Photography: Stereo photography in underwater archaeology has been applied primarily to mapping (Höhle, 1971; Bass, 1966; Bass, 1968; Green, 1971; Kodak, 1972). Stereo separation for photography of underwater objects is determined just as it is with terrestrial photography. Photography of underwater objects from above the surface of the water requires that

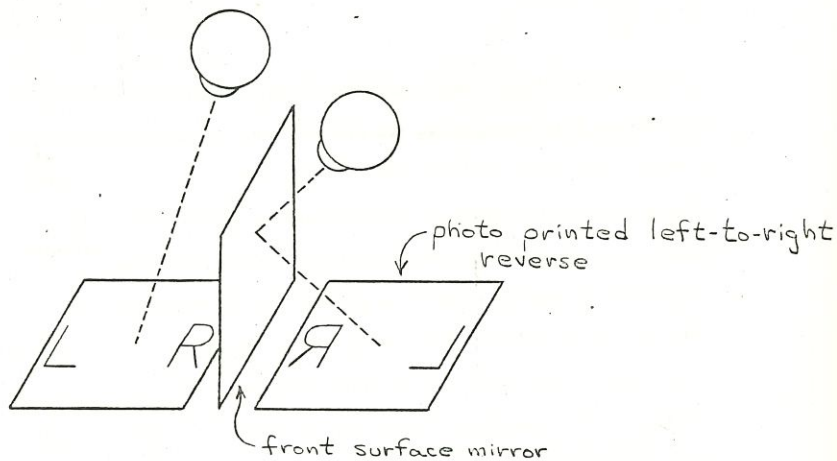


Figure 11: A single mirror stereo viewer

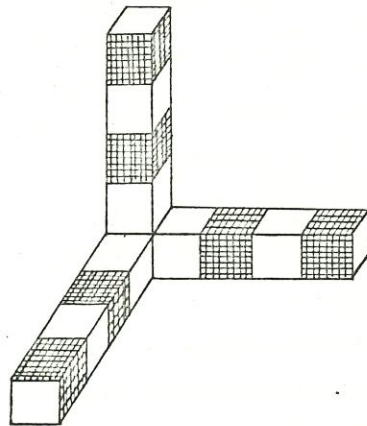


Figure 12: A 3-D photo scale

special camera filters be selected for greater visibility of deep objects (Colwell, 1960, p. 93; Helgeson, 1970; Mertens, 1970; Specht, 1973).

**Stereo Drawing:** Both contour maps and plans of sites can be presented as stereo pairs to clarify the dimension of depth in drawings (Gay, 1971; MacMahan, 1972; Gregory, 1970, p. 128, 168).

**Maps from Photos:** The preparation of accurate, three-dimensional maps from stereo photographs is a technical and detailed process (American Society of Photogrammetry, 1952; Hallert, 1960); it is easier, however, to make two-dimensional or plan maps. Both in architecture and archaeology, stereo photography has found application in the precise drawing of objects large and small (Carbonnel, 1965; McFadgen, 1971; Whittlesey, 1966; Maruyasu, 1965; Simpson, 1967; Schwarz, 1964; Wiltshire, 1967; Shmutter, 1970; Karara, 1972; American Society of Photogrammetry, 1971; Graham, 1972).

For this type of photography it is a good idea to include a three dimensional metric stick in the scene, with perpendicular rods as shown in Figure 12. For use in measurement, align the rods with the principal dimensions of the object of interest. With fairly good accuracy, measurements can be taken from single photos by having such a metric scale in your scenes (Janke, 1972; Williams, 1969; Williams, 1972).

**Substitutes for Stereo Photography:** There is a simple optical method of seeing and photographing contour lines of constant depth on any object. A light illuminates the object through a grating of parallel lines; at the same time the object is viewed through the grating. The correspondence of the shadow of the grating on the object with the image of the grating

forms a precise topographic map (Takasaki, 1970; Takasaki, 1973; Wasowski, 1970). This must be seen to be believed, but it can save considerable time in contouring, determining volumes, and recording subtle shapes.

Another technique which one should keep in mind is slit photography. With this technique a cylindrical object such as a vase is rotated in front of a special camera similar to a panoramic camera. The film is advanced in synchronism with the object's rotation and a thin line of the cylinder is scanned along the film. This allows the entire circumferential view of an object to be seen in one "undistorted" photograph. This camera, the R.E. Periphery Camera, is made by Research Engineers, Ltd., Orsman Road, London.

Holography, the recording of three-dimensional shapes with laser light, may eventually prove practical, although it is not yet (Smith, 1969, p. 23; Hammond, 1973).

Projecting stereo photos: Stereoscopic slides can be projected on screens by either the anaglyph method mentioned before or by using polarized light. Both methods require that each viewer wear special goggles (Kingslake, 1951, p. 197; Rule, 1941a; Rule, 1941b; Spottiswoode, 1953; Dewhurst, 1954).

Aerial Photography: Stereo photography is invaluable for aerial reconnaissance and mapping. Three large manuals from the American Society of Photogrammetry provide good introductions to this field (American Society of Photogrammetry, 1952, 1960; Smith, 1968). The history of aerial photography applied to archaeology is described in a book by Davel (1969).

Commercially available stereo photography equipment: There are several models of stereo cameras on the market now; most of these use a single camera equipped with two synchronized lenses separated by about 65 mm (Morgan, 1954; Kaiser, 1955). Stereo attachments are also available that can be put in front of the lens of a standard camera in order to split the normal frame into two stereo views. One good source of new and used stereo equipment is Olden Camera, 1265 Broadway, New York City 10001.

## PART II: THE BASIS OF STEREOSCOPIC PHOTOGRAPHY

### Cues for Depth Perception

Our perception of depth and distance is derived from a variety of visual information, called cues. While some of these cues require binocular vision, many are monocular. A single cue can sometimes give erroneous information (Ittelson, 1952), but a combination of cues increases reliability. Some of the major perceptual cues are listed below (Ogle, 1962, p. 251; Graham, 1966, p. 504). Some of them help to determine the absolute distance to an object while others indicate only the relative depth between objects.

**Relative Size:** The angular size of an object decreases in proportion to its distance, so that if we know the dimensions of an object, we can estimate its absolute distance from us. Linear perspective as typified by railroad tracks converging in the distance, is an example of this distance cue.

**Interposition:** Near objects can partially block the view of objects farther away. This cue does not allow one to measure either relative or absolute distance.

**Aerial Perspective:** The contrast of distant objects is decreased because the intervening air or water is not perfectly transparent. This cue is related to interposition and most commonly results from haze in the air.

**Light and Shade:** Highlight and shadow patterns reveal the relief of an object. This can be the primary depth cue in photographs of stone and terra-cotta figurines.

**Image Blurring:** If accommodation, the focus of the eye, allows one object to be seen sharply, another nearer or farther can be blurred.

In the language of photography, this is called the circle of confusion resulting when a point is not imaged at the camera's focal plane.

**Motion Parallax:** When one's head moves sideways, near objects appear to move relative to farther objects. While all of the above depth cues can be recorded on a single ordinary photograph, this one cannot except through the special technique of holography.

**Proprioception:** The muscles of the eyes which control accommodation and binocular convergence send signals to the brain which indicate object distance. This cue is surprisingly weak and unreliable for both absolute and relative distance estimation.

**Stereopsis:** The retinal disparity between one's pair of eyes is the best visual cue to relative depth. This cue to relative distance, which is a result of binocular parallax in a scene, tells one nothing about the absolute distance to an object.

### Binocular Disparity

Because of their separation of about 65 mm, each of our eyes sees a slightly different view of the same three-dimensional scene. The eyes converge to give the closest correspondence between this pair of images and then the brain interprets the remaining disparity as relative depth in the scene. Note that the word "parallax" usually describes the difference in a scene from two viewpoints while the word "disparity" usually describes the difference in the retinal image between one's pair of eyes; however, in most applications these words will be synonymous.

The measure of disparity is the difference in angular separation between corresponding points as seen from two separate locations; see

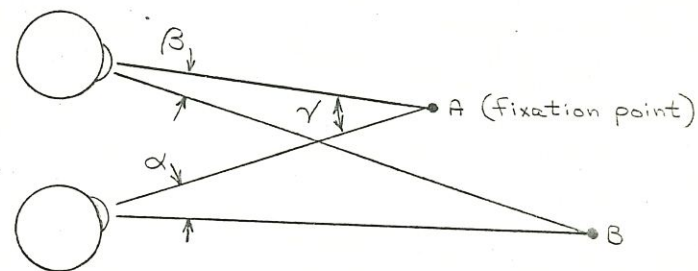
Figure 13. This disparity must exceed a minimum threshold to be perceived. That is to say, one has no stereoscopic depth perception of small objects at great distances. The difference between binocular parallax and the angle of binocular convergence is also illustrated in Figure 13.

Angular disparity changes with: distance to the scene, depth of the scene, and effective eye separation. With photography one controls effective eye separation with the spacing of the viewpoints of the photo; this is the stereo separation.

When retinal disparity is somewhat greater than the minimum threshold of stereoscopic perception, the brain's fusion of the binocular images causes them to be perceived as one three-dimensional scene (Pettigrew, 1972). As this disparity increases further, a maximum is reached and the two binocular images are too different to be fused. This phenomenon is called diplopia when images are doubled. It is easily experienced by looking at a distant object with a finger held up in front of one's eyes.

With a slight amount of diplopia good stereo vision is still easy and in fact so commonly experienced that it goes unnoticed. Further increase in diplopia allows one to estimate only roughly the relative proximity of targets. Even further increase yields only a confusion with no information about depth.

All of these limits to stereopsis increase with angle from the fixation point as indicated in Figure 14 (Ogle, 1962, p. 283; Le Grand, 1967, p. 217). Since this graph is for a typical observer, the limits can vary considerably (Ogle, 1950, p. 65).



$$\text{angular disparity between points A \& B} = |\alpha - \beta|$$

$$\text{binocular convergence} = \gamma$$

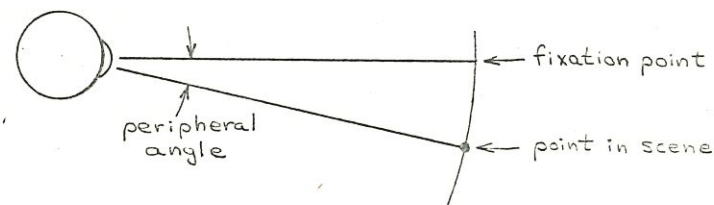
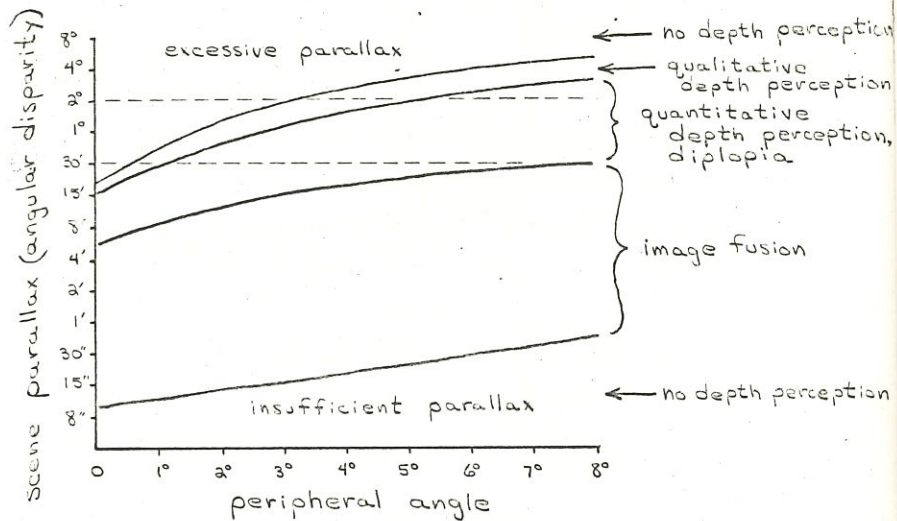


Figure 13: Some ocular angles



data from: (Ogle, 1962, p. 283) & (LeGrand, 1967, p. 217)

Figure 14: The limits of stereoscopic vision

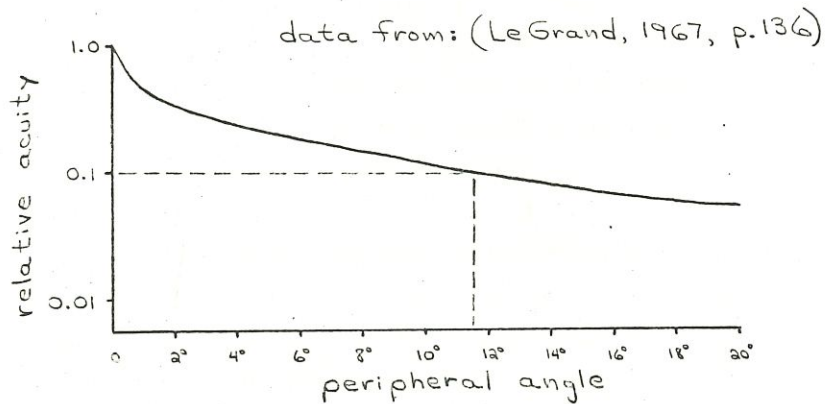


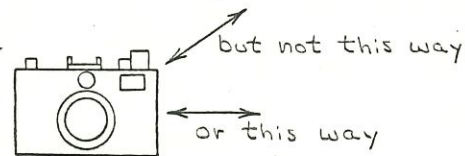
Figure 15: Visual Acuity


This graph should be interpreted in conjunction with the variation in visual acuity (Le Grand, 1967, p. 136) as shown in Figure 15. Note that acuity has decreased by a factor of ten at an angle of  $10^\circ$  from the eye's point of fixation. This indicates that one uses only the central portion of one's retina for accurate perception and the eye must get its detailed information by scanning a scene.

This figure suggests also that the depth of scene need only be considered as the largest depth found in any area within a  $20^\circ$  angle. For example, in a stairstepping or sloping scene greater than  $20^\circ$  wide, the effective visual depth of scene for the calculations of stereo separation can be less than the total depth of scene.

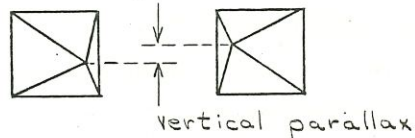
These two figures indicate that the  $2^\circ$  parallax chosen for stereo photography is a maximum practical limit. However, one may choose a greater or lesser parallax for specific applications. Earlier work by French (Judge, 1935, p. 28) indicated a maximum parallax of about  $0.5^\circ$  for stereo vision. If one wishes the best stereo representation, one will have to do some experiments to insure that one finds the optimum point where depth is apparent in the scene's areas of low relief and that excessive diplopia does not occur in the areas of high relief.

The above discussion has been on horizontal parallax between stereo photos. Incorrect alignment during photography or viewing can also add vertical parallax. This can be prevented by translating between stereo photo points parallel to one of the two directions of the camera's frame. Vertical parallax can also be eliminated later in printing or viewing, but this is difficult to do with color transparencies. Figure 16 illustrates the problem; not only does this vertical parallax not add

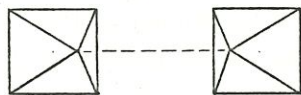


To prevent vertical parallax when taking stereo photos:  shift this way

If you translate obliquely between photo positions, you will get stereo photos like this:



When you should have gotten this:



This vertical parallax can be corrected by rotating each photo before viewing

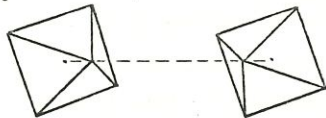


Figure 16: Vertical parallax

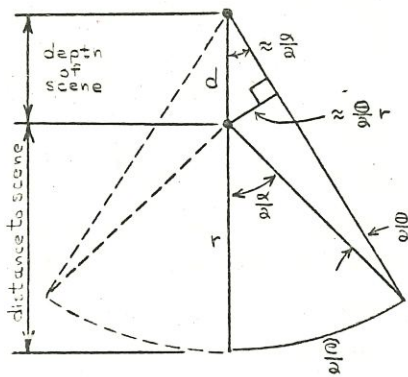
depth information, if it exceeds  $1^{\circ}$ - $2^{\circ}$  (Ogle, 1950, p. 66; Hallert, 1960, p. 55), stereopsis is impossible. Small amounts of vertical parallax can be corrected by the viewer's eyes with cyclorotation: each eye rotates about its axis of view.

When taking oblique stereo photos with a large angular separation, vertical parallax is prevented by aligning the camera in its two positions along a longitudinal line of an imaginary hemisphere centered on the scene. With a small stereo separation, camera alignment with a line of latitude can be used; this is usually easier to do and the resulting vertical parallax will be slight.

#### Geometric Calculations for Stereo Separation

There are several slightly different approaches to the determination of how the angular parallax of a scene varies with the separation of the viewpoints. Figure 17 illustrates the simplest analysis. Two points represent the near and far points in the scene. If one moves symmetrically around the scene along an arc centered on the near point in the scene, the far point appears to move through an angle called the angular parallax of the stereo separation. Because of the symmetry of this model, half of it indicates the geometry for this calculation of parallax.

A more accurate analysis is given in Figure 18. Two related derivations are given here: one assumes that the foreground is more important and keeps the near point in the scene centered by pivoting about it; the other assumes that the background is more important and has the far point in the scene as a pivot. Note that both approaches can be simplified to the approximation of Figure 17 for shallow scenes.



$\alpha$  = pivot angle about near point in scene  
 $\theta$  = arc length of stereo separation  
 $\Phi$  = angular parallax of scene

Assume  $\alpha$  and  $\Phi$  small so that  $\sin \frac{\alpha}{2} \approx \frac{\alpha}{2}$  &  $\sin \frac{\Phi}{2} \approx \frac{\Phi}{2}$   
 In radians,  $\frac{\alpha}{2} \approx \frac{\theta}{2r}$  and  $\frac{\Phi}{2} \approx \frac{\theta}{2r}$ ; therefore  $\frac{\alpha}{2} \approx \frac{\Phi}{2}$   
 and  $\frac{r}{d} = \frac{\theta}{d} \frac{r}{\theta}$  or  $\frac{r}{d} = \frac{\theta}{d} \left(\frac{r}{\theta}\right)^2$

If we want the parallax to be at most  $2^\circ$ , then  
 $\theta \leq 2^\circ = \frac{\pi}{90} \approx 0.04$  radian

So that the stereo separation formula is:  
 $\frac{r}{d} \leq 4\% \frac{r}{d}$  or  $\frac{r}{d} \leq 4\% \left(\frac{r}{d}\right)^2$

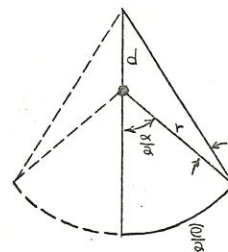
Several values from these formulas are:

$\frac{r}{d} = \frac{\text{distance to scene}}{\text{depth of scene}}$	1	$2\frac{1}{2}$	5	10	25
$\frac{r}{d} = \frac{\text{stereo separation}}{\text{distance to scene}}$	4%	10%	20%	40%	1
$\frac{\theta}{d} = \frac{\text{stereo separation}}{\text{depth of scene}}$	4%	25%	1	4	25
$\alpha = \text{pivot angle}$	$2^\circ$	$6^\circ$	$11^\circ$	$22^\circ$	$57^\circ$

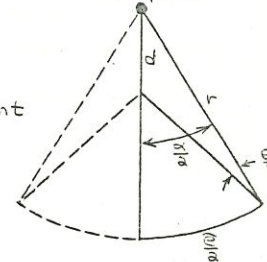
maximum values

Figure 17: Approximate analysis of stereo parallax

pivot about near point



pivot about far point



$\alpha$  = pivot angle  
 $d$  = depth of scene  
 $r$  = distance to pivot point  
 $\theta$  = pivot arc length  
 $\Phi$  = angular parallax

$$\frac{r \cos \frac{\alpha}{2} + d}{r \sin \frac{\alpha}{2}} = \cot \left( \frac{\alpha}{2} - \frac{\Phi}{2} \right)$$

$$\frac{r \cos \frac{\alpha}{2} - d}{r \sin \frac{\alpha}{2}} = \cot \left( \frac{\alpha}{2} + \frac{\Phi}{2} \right)$$

$$\frac{d}{r} = \sin \frac{\alpha}{2} \cot \left( \frac{\alpha}{2} - \frac{\Phi}{2} \right) - \cos \frac{\alpha}{2}$$

$$\frac{d}{r} = \cos \frac{\alpha}{2} - \sin \frac{\alpha}{2} \cot \left( \frac{\alpha}{2} + \frac{\Phi}{2} \right)$$

$$\therefore \alpha \approx \Phi \left( \frac{r}{d} + 1 \right)$$

$$\therefore \alpha \approx \Phi \left( \frac{r}{d} - 1 \right)$$

These last approximations give a maximum error of 3% at  $\alpha = 20^\circ$

These equations are tabulated below for  $\Phi = 2^\circ$

$\alpha$ , pivot angle	$3^\circ$	$4^\circ$	$6^\circ$	$10^\circ$	$15^\circ$	$20^\circ$	$30^\circ$	$40^\circ$
minimum r/d for pivot about near point in scene	0.5	1.0	2.0	4.0	6.5	8.9	13.9	18.5
minimum r/d for pivot about far point in scene	2.5	3.0	4.0	6.0	8.5	11.0	15.9	20.4

Figure 18: Pivoting about a stereo scene

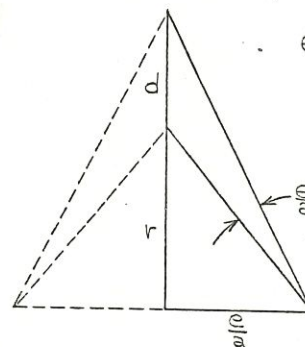
As one's distance from the scene to be photographed increases, it becomes easier to use a straight line separation between photo points. The analysis of the geometry of this problem is given in Figure 19. Once again, this exact analysis will yield a simple approximation in many cases. The exact analysis is tabulated in Figure 20 and plotted in Figure 4.

Several authors have determined stereo photo separations for specific types of scenes. (Symons, 1969, p. 1463; Wienert, 1960; McKay, 1948, p. 170; Kaiser, 1955, p. 117). Another author has developed a general formula similar to those above and has used an angular parallax of  $1^\circ$  (Judge, 1935, p. 38; Dalzell, 1936, p. 174). The similarity between some formulas for parallax is shown in Figure 21.

If one takes stereo photos of scenes with low relief by rotating the optic axis of the camera about the center of the scenes, it will be found that large angular rotations will cause excessive keystoneing. In Figure 22 the maximum swing angle is determined. As an example, a camera with a 50 mm lens and 35 mm film (24 x 36 mm format) should never be rotated over  $22^\circ$  in front of a scene with flat surfaces.

One can prevent keystone parallax in stereo photography by keeping the optic axis of the camera at the two photo positions parallel to each other. With 35 mm cameras it is convenient to move between stereo photos to give a two-thirds overlap in the photos; then a 24 mm square will be common to each frame in the stereo pair. While it is easy to determine the required stereo separation from an estimation of one third of the width of the scene in the camera's viewfinder, one can also determine the separation from the distance to the scene as shown in Figure 23.

All of the previous examples of parallax as a function of stereo separation in Figure 17 through 21 have assumed that the scenes had sharp edges. As



$\theta$  = angular parallax, the angle of relative displacement of the scene's near point relative to its far point.

$d$  = depth of scene

$r$  = distance to near point of scene

$e$  = stereo separation

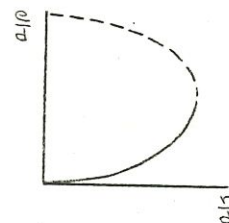
$$\frac{\theta}{a} = \tan^{-1} \frac{r+d}{e/a} - \tan^{-1} \frac{r}{e/a}$$

$$\tan \frac{\theta}{a} = \frac{(e/a)d}{(e/a)^2 + r(r+d)}$$

$$e = \frac{1}{\tan \frac{\theta}{a}} \left[ d \pm \sqrt{d^2 - 4r(r+d) \tan^2 \left( \frac{\theta}{a} \right)} \right]$$

$$\frac{e}{a} = \frac{1 - \sqrt{1 - 4 \frac{r}{d} \left(1 + \frac{r}{d}\right) \tan^2 \frac{\theta}{a}}}{\tan \frac{\theta}{a}}$$

$$\frac{r}{a} = \frac{\frac{e}{a}^2 - \sqrt{\left(\frac{e}{a}\right)^2 - 4 \left(1 + \frac{e}{a}\right) \tan^2 \frac{\theta}{a}}}{\tan \frac{\theta}{a}}$$

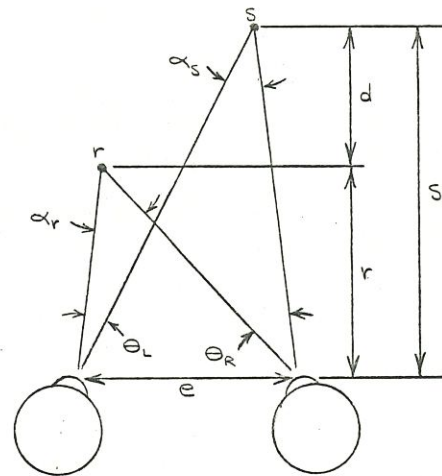


for  $r \gg d$  this simplifies to  $\frac{\theta}{a} \frac{r}{a} = \theta \frac{r}{a}$   
 for  $r \ll d$  this simplifies to  $\frac{\theta}{a} = \theta$

Figure 19: The parallax of stereo translation

$\frac{d}{r}$ distance to scene depth of scene	$\frac{e}{2d}$ maximum stereo separation twice depth of scene
0.1	0.00192
1	0.0349
2	0.105
3	0.211
4	0.352
5	0.529
6	0.743
7	0.995
8	1.286
9	1.617
10	1.989
11	2.405
12	2.867
13	3.376
14	3.936
15	4.551
16	5.224
17	5.962
18	6.770
19	7.656
20	8.632
21	9.710
22	10.91
23	12.26
24	13.79
25	15.59

Figure 20: Maximum stereo separation



$$\alpha_s \approx \frac{e}{s}, \alpha_r \approx \frac{e}{r}$$

$$\alpha_r - \alpha_s = e \left( \frac{1}{r} - \frac{1}{s} \right) = \theta_R - \theta_L = \theta$$

This is the equation of Fry (1965, p.75)

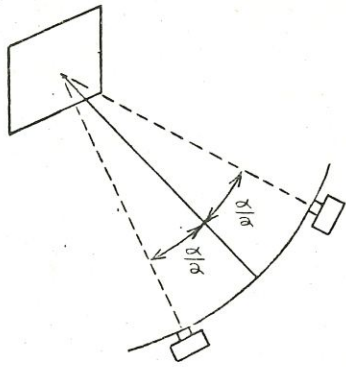
$$\frac{1}{e} = \frac{1}{\theta} \left( \frac{1}{r} - \frac{1}{s} \right)$$

$$e = \frac{1}{\frac{1}{r} - \frac{1}{s}} = \frac{\theta r s}{s - r} = \theta \frac{r}{d} (r + d)$$

The above is the equation of Colardeau (Judge, 1935, p.38) if  $\theta = 2\%$ , a parallax of  $1^\circ$ .

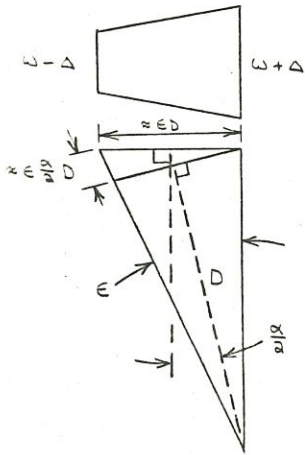
If  $r \gg d$ , then  $e = \theta \frac{r^2}{d}$ , which is the equation of Graham (1965, p.524) and the approximation of Fig. 17

Figure 21: Related parallax formulas



$\alpha$  = rotation angle  
 $w$  = height of scene  
 $\Delta$  = change in scene height due to keystoneing  
 $k$  = maximum size change between images which will still allow stereo vision

$\beta$  = angular height of scene  
 $\epsilon$  = angular width of scene  
 $D$  = distance to scene



For fusion,

$$\frac{\Delta}{w} \leq k = 15\%$$

$$w = \beta D$$

$$(w + \Delta) - (w - \Delta) = 2\Delta = \epsilon \beta \frac{\alpha}{\sin \alpha} D$$

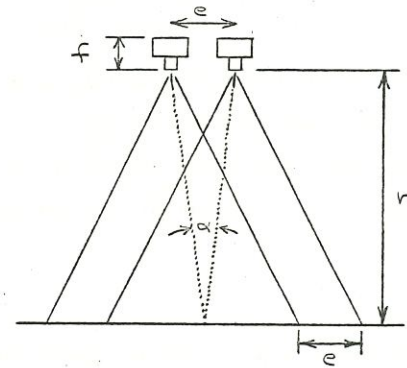
$$\therefore \frac{\epsilon \beta \alpha}{\sin \alpha} \leq k$$

$$\epsilon \alpha \leq 2k$$

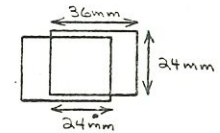
$$\alpha \leq \frac{2k}{\epsilon}$$

Example: 24x36 mm format, 50 mm focal length lens, and a low relief scene which fills frame  
 $\epsilon \approx \frac{36}{50}$ ,  $\max \frac{\alpha}{\epsilon} = \frac{0.15}{\frac{36}{50}} = \frac{7.5}{36} \approx \frac{1}{5}$  radian  $\approx 11^\circ$

Figure 22: Maximum angle of rotation when photographing flat scenes before keystoneing becomes excessive



$\alpha$  = effective perspective difference angle  
 $e$  = camera translation or separation  
 $f$  = image focal distance usually lens focal length



With 35mm film and  $\frac{2}{3}$  stereo overlap

$$\frac{e}{r} = \frac{12}{f}$$

Several values from this formula are listed:

f, mm	6	12	24	36	48	60	72	120
e/r	2	1	1/2	1/3	1/4	1/5	1/6	1/10
$\alpha$	90°	53°	28°	19°	14°	11°	10°	6°

Figure 23: Parallel camera alignment for flat scenes

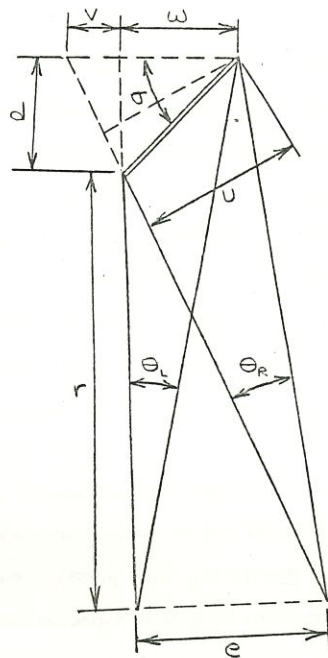
the scene slopes become flatter, a greater stereo separation can be used, even though the depth of the scene remains constant. Figure 24 illustrates the calculation of this effect; while the parallax decrease is small, it does give an extra safety margin to the 2° maximum parallax that has been chosen.

#### The Geometry of Perceived Space

Some writers on stereo photography have assumed that the distance to an object as perceived by viewing a three-dimensional scene or viewing stereo photographs is determined uniquely by the angle of convergence of the observer's eyes (Yacoumelos, 1972; Dewhurst, 1954, p. 31; Spottiswoode, 1953, p. 141). Experiment, however, has shown that subjective space is a distortion of the simple geometry of objective space, with the angle of convergence a poor depth cue (Ogle, 1962, p. 316; Graham, 1965, p. 520). An example of this is the fact that stereo photos can be viewed with a separation equal to the spacing of one's eyes. While the angle of binocular convergence will then be zero, the scene will not appear to be at an infinite distance (La Prade, 1972, p. 1178).

There have been many experiments for mapping our visual perception. For example, Gilinsky has shown that apparent distance is less than actual distance and that the difference increases with actual distance (Ogle, 1962, p. 256). Of greater interest for stereo photography is the variation of apparent depth with apparent distance (Fry, 1965, p. 75) and depth distortion as determined by the angles and distances of photography and viewing.

Stereo photography with camera separations greater than 65 mm is called hyperstereoscopy; it gives the viewer the effect of seeing a small nearby model of the real scene. In addition to the enhancement of depth perception which this effect allows, there can be an additional exaggeration of depth



$$\begin{aligned} \sigma &= \text{scene slope} \\ \Theta &= \Theta_R - \Theta_L = \text{parallax} \\ \frac{d\Theta}{d\sigma} &= \tan \sigma \\ \text{assume } \sigma &\approx 45^\circ, d \ll r \\ \Theta_L &= \frac{\sigma}{r} \text{ in radians} \\ \Theta_R &= \frac{v+w}{r^2+e^2}, \frac{v+w}{r} = \frac{r}{r^2+e^2} \\ \Theta_R &= \frac{(v+w)r}{r^2+e^2} = \left(\frac{de}{r} + w\right) \frac{r}{r^2+e^2} \\ \Theta &= \frac{\left(\frac{de}{r} + w\right)r}{r^2+e^2} - \frac{w}{r} \\ &= \frac{\left(\frac{de}{r} + w\right)r^2 - w(r^2+e^2)}{r(r^2+e^2)} \\ &= \frac{de - we^2}{r(r^2+e^2)} \end{aligned}$$

Substituting for  $w$ ,

$$\begin{aligned} \Theta &= \frac{de}{r(r^2+e^2)} \left(r - \frac{e}{\tan \sigma}\right), \Theta \uparrow \text{ as } \sigma \uparrow \\ \text{for } \sigma &= 90^\circ, \Theta = \frac{de}{r^2+e^2} \\ \text{for } \sigma &= 45^\circ, \Theta = \frac{de}{r^2+e^2} \left(1 - \frac{e}{r}\right) \end{aligned}$$

Figure 24: Scene slope and angular parallax

which alters the apparent slopes of the scene. This second effect is called a shape ratio (Spottiswoode, 1953, p. 37); it is the ratio of depth magnification to width magnification. It causes cubes to appear flattened or extended in the direction of viewing.

When a stereo pair of photographs is viewed with an angle of binocular convergence of  $11^{\circ}$ , this magnification of slope is approximately five times the ratio of stereo separation to the distance to the scene (La Prade, 1972; Yacoumelos, 1973). This indicates that slopes will be somewhat flattened when viewing photos of a scene photographed with an angle of convergence of the stereo camera less than  $11^{\circ}$ .

Depth exaggeration occurs also when one views an actual scene. Here, the separation of one's eyes remains constant; it is found that the apparent slopes of scenes decrease with distance (La Prade, 1973, p. 38). Therefore, depth exaggeration when viewing stereo photos is possibly similar to that of viewing real scenes.

A surface around one's direction of fixation whose points appear by stereopsis to be the same distance from the observer is called a horopter (Ogle, 1962, p. 352). This surface is not usually a plane perpendicular to the observer's line of sight. Instead, Kroncke has found that this surface is usually concave toward the observer for viewing distances less than about one meter and concave away from the observer for greater distances (Le Grand, 1967, p. 237).

A similar nonlinearity exists in a direction along one's line of sight. In what is called the alley experiment, Blumenfeld asked his subjects to arrange two lines of lights so that they formed apparently parallel lines extending directly away from them (Graham, 1965, p. 538). The actual arrangement of the lights was found to be a pair of diverging curved lines.

These relationships between objective and subjective visual space have been summarized and combined with mathematical transforms (Luneburg, 1947). However, these equations appear to imply a greater accuracy than the meager experimental data contain (Ogle, 1962, p. 395).

The ancient Greek architects evidently were familiar with the correction of subjective visual distortions. While it is possible that the idea of entasis was a remnant of much earlier construction (Marquand, 1909, p. 89), there are a number of other independent curvilinear deviations from straight and perpendicular lines found in Greek architecture. Examples of this are the curvature of the stylobate and entablature and the upward convergence of the axes of columns in Greek temples (Anderson, 1927, p. 119). While it is possible that these variations were simply to make the forms pleasing visually, it is also possible that a definite attempt was made to create straight and perpendicular lines in the viewer's subjective perception.

When viewing stereo photos, it sometimes happens that their depth information is overridden by one's prior experience. For example, a hollow mask of a human face can sometimes look like a normal face (Gregory, 1970, p. 128). A related effect is due to the fact that one is used to seeing shadows cast below an object or toward the observer; when stereo photos are viewed with the shadows in the photos falling away from the viewer, relief can be reversed so that depressions become elevations (Howard, 1970, p. 149; American Society of Photogrammetry, 1952, p. 527). In some fairly artificial stereo views, stereopsis can give only ambiguous depth information (Julesz, 1969, p. 591; Julesz, 1971, p. 186).

#### Testing Stereoscopic Vision

Roughly 10-15% of the population has weak stereo vision. While it is

difficult to detect deficient stereo vision when normal photographic scenes are viewed with a stereoscope, one published test example allows a good qualitative evaluation of stereopsis (Avery, 1968, p. 33). Other, quantitative, stereopsis tests allow comparisons between individuals (Howard, 1970, p. 150; Hallert, 1960, p. 59; Julesz, 1971, p. 270). Stereoscopic acuity can be improved by training; it also usually improves during a stereo viewing session (Howard, 1970, p. 143).

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